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# KKD-7104 Seat No.________

## B. Sc. (Sem. I) Examination

## November / December - 2014

## CCMAT - 111 : Mathematics

*(New Course)*

Time : 3 Hours] [Total Marks : 70

Q:1 (a) State and prove Leibnitz's theorem. (8)

OR

(a) State and prove Cauchy's mean value theorem.

(b) Attempt any Two: (10)

(1) If $y=(\sin^{-1}x)^2$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n = 0$.

(2) If $f(x)=x^{m+n}$ then prove that, $\frac{f^{(n)}(1)}{n!}=\binom{m+n}{n}=\sum_{r=0}^{n}\binom{m}{r}\binom{n}{r}$, $m,n\in N$, $m>n$.

(3) Prove that $Log(1-x^2)=-x^2-\frac{x^4}{2}-\frac{x^6}{3}-\dots\dots$

Q:2 (a) Prove that formula of length of arc is, $S=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$. (8)

OR

(a) Find the formula of $\int_0^{\pi/2}\sin^m x.\cos^n x\,dx$, m, n $\in$ N.

(b) Attempt any Two: (10)

(1) Prove that : $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\dots\dots\left(1+\frac{n}{n}\right)\right]^{1/n}=\frac{4}{e}$.

(2) Find the length of arc of $y^2=4ax$ at vertex A(0,0) to end point of latus rectum.

(3) Find the surface area generated by one arc of $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about x-axis.

Q:3(a) Prove that $\left(\bar{a}+\bar{b}\right).\left\{\left(\bar{b}+\bar{c}\right)\times\left(\bar{c}+\bar{a}\right)\right\}=2[\bar{a}\ \bar{b}\ \bar{c}]$, where $\bar{a}, \bar{b}, \bar{c} \in R^3$. (8)

OR

(a) Obtain the polar equation of a straight line passing through the points $(r_1,\theta_1)$ and $(r_2,\theta_2)$.

(b) Attempt any Two: (10)

(1) Find out reciprocal vector set of the set {(4,1,2),(2,-1,1),(-1,-1,1)}.

(2) If $\bar{F}=(x,y,z) = (x^2z, -2y^3z^2, xy^2z)$ then find out $div\bar{F}$ at the point (1,-1,1).

(3) Obtain the polar equation of the straight line having its parametric equations are $x=3+7t$, $y=2+t$, $t\in R$. Also determine P and $\alpha$.

Q:4(a) Attempt any Two : (8)

(1) Find the equation of the sphere touching co-ordinate plane and passing through point (1,5,4).

(2) Discuss the geometrical representation of an equation, $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ in $R^3$.

(3) Find the equation of the tangent planes to the sphere $x^2+y^2+z^2-4x-2y-4=0$, parallel to the plane $2x-y-z=1$.

(b) Attempt any Two: (8)

(1) Obtain the condition that the homogeneous equation, $ax^2+by^2+cz^2+2fyz+2gzx+2hxy = 0$ represents a right circular cone. Also find its axis and semi-vertical angle.

(2) Find the equation of a cylinder whose axis is $\frac{x}{1}=\frac{y}{-1}=\frac{z}{-1}$ and the guiding curve is $2x^2+3y^2=1$, $z=0$.

(3) Find the equations of the tangent planes to the ellipsoid $x^2+4y^2+3z^2=10$ passing through the straight line $3x+4y+9z=20$, $x=3z$.