GAZ-467-68 Seat No.__________

B. Sc. (Sem. VI) Examination

March / April – 2017

Mathematics : CC-MAT-603 (A) & (B)

(A) : Topology

(B) : Number Theory

Time : **3** Hours] [Total Marks : **70**

(A) : Topology

Instructions :

(1) All questions are compulsory; there are five question.

(2) Figures to the **right** indicate marks of the corresponding question.

1 (a) Show that a subset O of a topological space is open is open if and only if O is a neighborhood of each of its points. **18**

(b) Prove that A is closed iff $A = \overline{A}$; where A is a subset of a topological space.

(c) Let

$$T = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$$

be a topology on $X = \{a, b, c, d, e\}$.

Determine the closure of the sets $\{a\}$; $\{b\}$ and $\{c, e\}$.

OR

1 (a) Let X be a topological space and let F be a closed set in X and let $A \subset F$. Show that $F \supset \bar{A}$. **18**

(b) Let X be a topological space.

Show that for each point $x \in X$ and each pair N, M of neighborhoods of x; $N \cap M$ is also a neighborhood of x.

(c) List all topologies on $X = \{a, b, c\}$ which consist of exactly four members.

2 (a) If A be a subset of a topological space; then show that $Int.(A) = \bigcup_{\alpha \in I} O_\alpha$; where $\{O_\alpha\}_{\alpha \in I}$ is the family of all open sets contained in A. **18**

(b) Show that

A function $f:(X,T) \to (Y,T')$ is continuous if and only if for each open subset O of Y; $f^{-1}(O)$ is an open subset of X.

(c) Show that the closed interval $A = [a, b]$ is homeomorphic to the closed unit interval $I = [0, 1]$.

OR

2 (a) Let $f:(X,T) \to (Y,T')$ be continuous at $a \in X$ and let $g:(Y,T') \to (Z,T'')$ be continuous at $f(a) \in Y$. Then show that $gof:(X,T) \to (Z,T'')$ is continuous at $a \in X$. **18**

(b) Let (X, T) be a topological space and $A \subset X$.

Show that $Bdry.(A)$ is closed and

$Bdry.(A) = Bdry.(C(A))$.

(c) Let $T = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}, X\}$ be a topology on $X = \{1, 2, 3, 4\}$.

Define $f : X \to X$ by

$f(1) = 2, f(2) = 4, f(3) = 2, f(4) = 3$. Show that f is continuous at 4; but not continuous at 3.

3 (a) Let Y be a non emtpy subset of a topological space X and let $T' = \{O' \subset Y / O' = O \cap Y;$ where O is an open set in $X\}$. Show that (Y, T') is a topological space. **18**

(b) Let X and Y be topological spaces and let $f : X \to Y$ be continuous. Show that if A is a connected subset of X; then f(A) is a connected subset of Y.

(c) Let $T = \{X, \phi, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on $X = \{a, b, c, d, e\}$. Find all connected subsets of X.

OR

3 (a) State and prove intermediate - Value Theorem. **18**

(b) If A be a connected subset of a topological space X and let $A \subset B \subset \overline{A}$; then show that B is also connected.

(c) Let X denote a two-point space in the indiscrete topology. Is X connected ? Why ?

4 Attempt any two : **8**

(a) State and prove fixed-point thorem.

(b) Show that $f : Y \to X$ is continuous iff $f' : Y \to f(Y)$ is continuous.

(c) Let A be a subset of a topolgical space. Show that if $x \notin \overline{A}$; then $x \notin F$ for some closed set F containing A.

5 Attempt any two : **8**

(a) Let X be a set; let $T_c = \{U \subset X / X - U$ either is countable or is all of X$\}$. Then show that (X, T_c) is a topological space.

(b) Let $X = \{a, b, c, d, e\}$ with topology $T = \{\phi, X, \{a\}, \{b, c, d, e\}\}$ and let $Y = \{a, c, d\} \subset X$. Then find out all open sets in Y and all closed sets in Y.

(c) Show that $A = [0, 1] \cup (2, 3)$ is a disconnected subset of R.

(B) : Number Theory

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate the marks of the corresponding question.

1 (a) State and prove the linear Diophantine equation. **6**

OR

(a) For positive integer a and b prove that

$\gcd(a,b).lcm(a,b) = ab$.

(b) Attempt any three : **12**

(1) Solve the Diophantine equation $158x - 57y = 7$.

(2) Prove by Mathematical induction

$$1^2 \cdot 2 + 2^2 \cdot 3 + 3^2 \cdot 4 + ... + n^2(n+1)$$

$$= \frac{n(n+1)(n+2)(3n+1)}{12}.$$

(3) For $n \geq 1$, prove that $6 / n\left(7n^2 + 5\right)$.

(4) Find the integers x and y satisfying

$\gcd(213, 121) = 213x + 121y$ by Euclidean Algorithm.

2 (a) The Linear Congruence $ax \equiv b \pmod{n}$ has a solution if and only if d / b, where $d = (a, n)$. **6**

OR

(a) For arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same nonnegative remainder when divided by n.

(b) Attempt any three : **12**

(1) Find the last two digit of the number 9^{9^9}.

(2) Find the remainder when $2017^{1969} + 1969^{2017}$ number divided by 12.

(3) Solve : $x \equiv 5 \pmod{11}, x \equiv 14 \pmod{29}$ and $x \equiv 15 \pmod{31}$ by use of Chinese Remainder theorem.

(4) Prove that there are infinitely many primes.

3 (a) State and prove Euler's theorem. **6**

OR

(a) State and prove Fermat's theorem.

(b) Attempt any three : **12**

(1) Verify that $\phi(n) = \phi(n+1) = \phi(n+2)$, where $n = 5186$.

(2) Find the remainder when 1!+2!+3!+.....+100! is divisible by 24.

(3) If p and $p+2$ are both twin prime then prove that $4((p-1)!+1)+p \equiv 0 \pmod{p(p+2)}$.

(4) Find last two digits of 3^{256} in its decimal represents.

4 Attempt any four : **16**

(1) Solve the linear congruence $25x \equiv 15 \pmod{29}$.

(2) Prove that $41 \mid 2^{20}-1$.

(3) $[a,b,c](a,b,c)=abc$ is true or not ? Justify your answer.

(4) Define Euler's Phi function and find the value of $\phi(5040)$.

(5) If a and b are given integers, not both zero, then a and b are relatively prime if and only if there exist integers x and y such that $1=ax+by$.

(6) Converse of the Wilson's theorem is true ? Justify your ans.