GAY-411 Seat No. ________

B. Sc. (Sem. IV) Examination

March / April – 2017

Mathematics : CC - MATH - 402

(Advanced Linear Algebra)

Time : Hours] [Total Marks : **70**

Instruction : All questions are **compulsory**.

1 (a) Explain Linear Transformation with a given matrix. **6**

OR

(a) For $A, B, C \in M_n$ and Scalar α, prove that **6**

(i) $A(BC) = (AB)C$

(ii) $\alpha(AB) = A(\alpha B)$

(b) Attempt any two : **12**

(1) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a Linear Transformation, where $T(a,b,c,d) = (a-2b+c, 2a+b-d, -3a+c+2d)$ and B_1, B_2 are standard bases of $\mathbb{R}^4$ and $\mathbb{R}^3$ respectively. Find $[T : B_1, B_2]$

(2) Let $A = \begin{bmatrix} -1 & 5 \\ 2 & 7 \\ 3 & -2 \end{bmatrix}$ and $B_1 = \{(1, 1), (-2, 5)\}$,

$B_2 = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ be ordered based of $\mathbb{R}^3$ and $\mathbb{R}^4$ respectively. Obtain the associated linear transformation T for A w.r. to bases B_1 and B_2.

(3) Find rank of a matrix $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 11 & -5 \\ 2 & -5 & 3 \\ 4 & 1 & 1 \end{bmatrix}$.

2 (a) Define linear functional with illustration. **6**

OR

(a) State and prove Triangular inequality for norm. **6**

(b) Attempt any two : **12**

(1) Let the linear basis $B = \{(1, 2), (5, -1)\}$ of Euclidean space $\mathbb{R}^2$. Find it's orthonormal basis.

(2) Applying Gram-Schmidt Orthonormalization process, obtain an Orthonormal set S' from a given L.I. set S in an inner product space U such that $[S] = [S']$.

Where $U = \mathbb{R}^4, S =$

$\{(1, -1, 0, 1), (2, 3, 5, 0), (7, 8, 0, 0)\}$.

(3) Let $B=\{(0,1,1),(1,0,1),(1,1,0)\}$ be a basis of $\mathbb{R}^3$ and a given functional $f\left(\in \mathbb{R}^3\right)$, $f(a,b,c)=ab+bc+ca$. Find it's dual basis.

3 (a) Define eigen vectors. Solve the system of Linear equations. 6

$$2x-3y=5$$
$$x+4y=9$$

OR

(a) Define characteristic equation. Solve the system of linear equations. 6

$$x+y+3z=4$$
$$x+3y-3z=-3$$
$$-2x-4y-4z=7$$

(b) Attempt any two : 12

(1) Let $A=\begin{bmatrix} 7 & 8 \\ -1 & 3 \end{bmatrix}$ find A^{-1}.

(2) Let $A=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ find eigen values and eigen vectors.

(3) Let $A=\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$ find A^{-1} using Cayley-Hamilton theorem.

4 Attempt any four : **16**

(a) If $A=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$ then find A^2.

(b) Let $A=\begin{bmatrix} 5 & 1 & 2 \\ -3 & 0 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ be a matrix. Prove that row rank of A=column rank of A.

(c) For $x, y \in U$ and scalars α, β

Prove that $4\langle x, y\rangle = \|x+y\|^2 - \|x-y\|^2$.

(d) Obtain an orthonormal basis from a given Linear basis B in an inner product space U.

Where $U=\mathbb{R}^3$, $B=\{(1,1,0)\,(2,5,-1),(0,3,2)\}$.

(e) Let $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ be a matrix. Find A^{-1} using Cayley-Hamilton theorem.
