GAY-409 Seat No. ________

B. Sc. (Sem. IV) Examination

March / April – 2017

Mathematics : CC - MATH - 401

(Advanced Calculus)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are **compulsory**.
(2) The figures to the **right** indicate the marks of the corresponding questions.

1 (a) For curve $y = f(x)$, prove that the radius of curvature $\rho = \dfrac{ds}{d\psi} = \dfrac{\left(1+y_1^2\right)^{\frac{3}{2}}}{y_2}$. **5**

OR

(a) Prove that $\beta(m,n) = \dfrac{(m-1)!\,(n-1)!}{(m+n-1)!}$, $m,n \in N$. **5**

(b) Attempt any three : **15**

(1) Find $S-\psi$ equation of curve

$x = a\cos^3\theta,\ y = a\sin^3\theta$.

(2) Find the double points of

$x^4 + y^4 - 18\left(x^2+y^2\right) + 81 = 0$.

(3) Prove that

$$\int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} dx = a^{-m} \cdot b^{-m} \cdot \beta(m,n).$$

(4) Prove that

$$\int_0^\infty \sqrt{y} \cdot e^{-y^2} dy = \frac{1}{2} \overline{\left| \frac{3}{4} \right.}.$$

2 (a) Prove that $\iint_S \frac{y^2 - x^2}{\left(x^2 + y^2\right)^2} dx\, dy$ does not exist **5**

where $S = [0,1] \times [0,1]$.

OR

(a) Evaluate $\iint_S x^2 y^2 dxdy$, where S is a region **5**

bounded by $y = 2x$, X-axis, $x = 4$.

(b) Attempt any three : **15**

(1) Using double integral, find the area of $x^2 + y^2 = a^2$.

(2) Change the order of $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} f\, dx\, dy$.

(3) Using Polar co-ordinate, evaluate

$$\iint_S \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dx\, dy,$$

$$S = \left\{(x,y) / x^2+y^2 \le 1, x \ge 0, y \ge 0\right\}.$$

(4) Evaluate $\int_0^a \int_{y=0}^{y=\sqrt{a^2-x^2}} \int_{z=0}^{b} \left(y^2+z^2\right) dx\, dy\, dz$.

3 (a) State and prove divergence theorem. 5

OR

(a) State and prove Green's theorem. 5

(b) Attempt any three : 15

(1) Prove that :

$$Curl\left(\phi \overline{f}\right) = \phi\, Curl\, \overline{f} + (grad\, \phi) \times \overline{f}.$$

(2) Verify Green's theorem.

$\oint_C \left[\left(xy+y^2\right)dx + x^2 dy\right]$ where C is boundry of region bounded by $y = x$ and $y = x^2$.

(3) Verify Stoke's theorem for

$\overline{f} = \left(-y^3, x^3, 0\right)$ where

$$S = \left\{(x,y) / x^2+y^2 \le 1, z = 0\right\}.$$

(4) Verify the divergence theorem

$$\iint_S \left[\left(x^3 - yz \right) dy\, dz - 2x^2 y\, dz\, dx + z\, dx\, dy \right].$$

Where S is a surface of the cube with faces $x = 0, x = a, y = 0, y = a, z = 0, z = a$.

4 Attempt any five : **10**

(1) Find the radius of curvature of

$x^2 + 2y^2 - 2x + 4y = 5$ at $(1,1)$.

(2) Prove that

$$\overline{|n} = 2 \int_0^\infty e^{-x^2} \cdot x^{2n-1} dx.$$

(3) Evaluate $\displaystyle\int_0^\infty \frac{dx}{1+x^4}$.

(4) Evaluate $\displaystyle\int_0^a \int_0^a xy \left(x^2 + y^2 \right) dx\, dy$.

(5) Evaluate $\displaystyle\int_0^5 \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} r^4 \cdot \sin\phi\, dr\, d\theta\, d\phi$

(6) If $\overline{r} = (x, y, z), |\overline{r}| = r, \overline{a} = \left(a_1, a_2, a_3 \right)$ then prove that $div\left(\overline{a} \times \overline{r} \right) = 0$.

(7) Prove that

$$\iint_S \left(Curl\, \overline{F} \right) \cdot \overline{n}\, ds = 0.$$