GCP-1224 Seat No. ________

B. Sc. (Sem. II) Examination

April / May - 2017

Mathematics : CC MAT - 122

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicates the marks of the corresponding question.

1 (A) State and prove De' Moivre's theorem. **7**

OR

(A) If q is any positive integer then $(\cos\theta + i\sin\theta)^{1/q}$ has q and only q different value.

(B) Attempt any **two** : **8**

(1) Prove that

$$\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^5} = \sin(4\alpha + 5\beta) - i\cos(4\alpha + 5\beta).$$

(2) Prove that

$$\frac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$$

(3) If $2\cos\theta = x + \frac{1}{x}$ then prove that

$$2\cos r\theta = x^r + \frac{1}{x^r}$$

2 (A) State and prove De' Alembert Ratio test 7

OR

(A) State and prove Cauchy Root Test.

(B) Attempt any **two** : 8

(1) If $(a+ib)^p = m^{x+iy}$ then prove that

$$\frac{y}{x} = \frac{2\tan^{-1}\left(\frac{b}{a}\right)}{\log\left(a^2+b^2\right)}$$

(2) Discuss the convergence

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots$$

(3) Prove that $\sinh^{-1} z = \log\left(z+\sqrt{z^2+1}\right)$

3 (A) Prove in usual notation 7

$$\frac{1}{f\left(D^2\right)} Sinax = \frac{1}{f\left(-a^2\right)} Sinax,\ f(a) \neq 0$$

OR

(A) Define linear differential equation and write the method of solving it.

(B) Solve any two differential equations. 8

(1) $\left(D^3+D^2-D-1\right)y = \cos 2x$

(2) $y+px = x^4p^2$.

(3) $\left(D^4-1\right)y = e^x \cos x$

4 (A) If A and B are $m \times n$ and $n \times p$ matrices respectively then prove that $(AB)^T = B^T A^T$. 7

OR

(A) Define :

(i) Hermitian matrix

(ii) Identity matrix

(iii) Transpose of a matrix

(iv) Symmetric matrix

(B) Attempt any **two** : 8

(1) Find the rank k of matrix $A = \begin{bmatrix} 3 & 2 & 0 & -1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$ using row reduction method.

(2) Solve the equation $x - y + 3z = 1$, $2x + y - z = 2$, $3x - y + 2z = 1$.

(3) Find A^{-1} for the matrix $A = \begin{bmatrix} 7 & 6 & 2 \\ 1 & 4 & 9 \\ 3 & 1 & 8 \end{bmatrix}$

5 Attempt any **five** : 10

(1) Show that $\arg(z_1 . z_2) = \arg z_1 + \arg z_2$.

(2) If $z = x + iy$ and $\bar{z} = x - iy, x, y \in R$ are complex conjugates then $\mathrm{Re}(z) = \frac{1}{2}(z + \bar{z})$ and $\mathrm{Im}(z) = \frac{1}{2}(z - \bar{z})$.

(3) Discuss the convergence of $\sum \frac{3^n n!}{n^n}$.

(4) Discuss the convergence of $\sum \frac{1+2+3+...+n}{n!}$

(5) Solve : $\left(D^3+2D^2+2D+1\right)y=0.$

(6) Solve : $yp = xp^2 + a.$

(7) If $A=\begin{bmatrix} 2 & 3 & -1 \\ -1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix}$ then find $adjA.$
