P.S.Science and H.D.Patel Arts College, Kadi.

Internal Examination

B.Sc. Sem: VI

Mathematics:CC-MATH-602(Analysis)

Date: 14/03/2017 Time : 2 Hours

1.(a)Suppose f is a continuous one to one mapping of a compact metric space X onto a metric space Y then the inverse mapping f^{-1} defined by $f^{-1}(f(x)) = x, (x \in X)$ on Y is also a continuous mapping of Y on to X.

(b) Show that continuous image of connected set is connected.

(c) Let $X = (0,1]$. Show that $f : X \rightarrow R, f(x) = \frac{1}{x}$ is continuous but not uniformly continuous.

OR

1.(a)Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

(b) State and prove Taylor's theorem.

(c) Evaluate (i) $\lim_{x \to 0} (1-x)^{\frac{1}{x}}$ (ii) $\lim_{x \to 0} \sin(\log x)$

2.(a) If $f_1 \in R(\alpha)$ & $f_2 \in R(\alpha)$ on [a,b] then prove that

(i) $f_1 + f_2 \in R(\alpha) on \ [a,b]$

(ii) $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 \, d\alpha + \int_a^b f_2 \, d\alpha$

(b) State & prove the Fundamental theorem of calculus.

(c) Define $\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$,where a is fixed, $f \in R[a,b]$

Using this definition prove that $\int_0^{\infty} \frac{\cos x}{1+x}\,dx = \int_0^{\infty} \frac{\sin x}{(1+x)^2}\,dx$

OR

2(a) If f is monotonic on $[a,b]$, and if α is continuous on $[a,b]$, then $f \in R(\alpha)$

(b) Prove that $\underline{\int_a^b} f\,d\alpha \le \overline{\int_a^b} f\,d\alpha$

(c) Suppose α increases on $[a,b]$, $a \le x_0 \le b$, α is continuous at x_0, $f(x_0)=1$ *and* $f(x)=0$ *if* $x \ne x_0$ then prove that $f \in R(\alpha)$ and $\int_a^b f\,d\alpha = 0$

3(a) Prove that $C(X)$ is complete metric space

(b) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$, for $n = 1,2,3,\ldots$ and suppose $f_n \to f$ uniformly on $[a,b]$ then prove that $f \in R(\alpha)$ on $[a,b]$ and $\int_a^b f\,d\alpha = \lim_{n\to\infty}\int_a^b f_n\,d\alpha.$

(c) Prove that sequence does not converges uniformly on R, where $f_n(x) = \dfrac{nx}{1+n^2x^2}$, $x \in R$ and $n = 1,2,3,\ldots$

OR

3 (a) If K is a compact metric space if $f_n \in C(K)$ $n = 1,2,3,\ldots$ and if $\{f_n\}$ convergence uniformly on K then $\{f_n\}$ is equi-continuous on K

(b) Prove that the series $\sum f_n(x)$ of functions defined on E convergence uniformly on E if and only if

for every $\varepsilon > 0$, $\exists$ a positive integer N such that

$$m \geq n \geq N \Rightarrow \left|\sum_{k=n}^{m} f_k(x)\right| < \varepsilon,\ x \in E$$

(c) If $f_n(x) = \dfrac{1}{1+nx}$, $x \in (0,1)$ then show that $f_n(x) \to 0$