PRAMUKH SWAMI SCIENCE AND H D PATEL ARTS COLLEGE,KADI

B. Sc. Semester VI (MATHEMATICS)

PAPER : CC 601 (ABSTRACT ALGEBRA)

[DATE : 11/03/2017 INTERNAL EXAM TIME : 2 Hours]

Q-1 (a) Define a ring

Give an example of

(i) A Ring which is not an integral domain

(ii) An integral domain which is not a field

(iii) A non commutative ring with a finite number of elements

[6]

OR

Q-1 (a) Show that the characteristics of a ring R with unity is n if and only if n is the smallest positive integer with $(n \bullet 1) = 0$ [6]

Q-1 (b) Attempt Any Two **[8]**

(a) Show that the characteristics of an integral domain is either a prime number or zero .

(b) Show that a field has no proper ideal

(c) For a given prime p, prove that $(Z_p, +; .)$ has no zero divisors

Q-2 (a) State and prove Gauss's Lemma [5]

OR

Q-2 (a) In usual notations, for non zero polynomials $f, g \in D[x]$, prove that $[fg] = [f] + [g]$ [5]

Q-2(b) : Attempt any TWO : [8]

(a) Discuss the irreducibility of a polynomial $8x^3+6x^2 - 9x + 24$ over Q

(b) If the degree of a polynomial $f(x) \in F[x]$ is n, then f(x) has at most n distinct zeros in F.

(c) Suppose f = (0,1,3,1,0,0.....) and g = (-1,0,-3,1,0,0,0.......) then find f+g and fg where $f, g \in Z[x]$.

Q-3 (a) Prove that an ideal M in a commutative ring R with unity is a maximal ideal if ring R/M is a field . [5]

OR

Q-3 (a) Let (R; +; .) be a ring with unity. Show that the mapping

$\emptyset : (Z; +; .) \rightarrow (R; +; .)$ where $\emptyset$ (n) = n 1 , n ε Z is a homomorphism with $K_{\emptyset}$ = <m> if the characteristic of ring R is m. [5]

Q-3 (b) Attempt any Two **[8]**

(a) Show that intersection of two ideals in a ring R is also an ideal in a ring R. What can u say about the union ? justify your answer.

(b) Suppose R is a ring with unity. If, homomorphism $\emptyset : (R; +; .) \rightarrow (R' ; \oplus ; \odot)$ with $\emptyset(1) \neq 0$, then show that $\emptyset$ (1) is a unity element of ring $\emptyset$ (R).

(c) Give an example of a prime ideal which is not a maximal ideal

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