P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - IV

10-3-2017] **Mathematics - 401** **[1-30 to 3-30**

(Advanced Calculus)

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1. [A] Obtain the radius of curvature of a curve $x = f(t)$, $y = g(t)$

OR

[A] Prove that $\overline{)n}\ \overline{)n+\frac{1}{2}} = \frac{\sqrt{\pi}\ \overline{)2n}}{2^{2n-1}}$

[B] **Attempt any two**

(i) Evaluate $\int_0^1 \sqrt{x}\,\sqrt[3]{(1-x^2)}\,dx$

(ii) Find the centre of curvature of curve

$x = a\,(\cos\theta + \log\tan\frac{\theta}{2})$, $y = a\sin\theta$.

prove that its evolute is catenary

(iii) If ϱ_1 and ϱ_2 are the radii of curvature at the extremities of the tocal chord of parabola $\frac{a}{r} = 1 + \cos a$, then prove

that $\frac{1}{\varrho_1^{\frac{2}{3}}} + \frac{1}{\varrho_2^{\frac{2}{3}}} = \frac{1}{a^{\frac{2}{3}}}$

2. [A] Let the function $f : S \to R$ be continuous on $S \subset R^2$ which closed and bounded by curves $x = a$, $x = b$, $y = \phi(x)$ and $y = \Psi(x)$, Where Ψ and Ψ are continuous function on [a, b] such that $\Psi(x) < \phi(x)$, $\forall x \in [a, b]$ then prove that

$$\iint_s f\,dxdy = \int_a^b \int_{\psi(x)}^{\psi(x)} f\,dxdy$$

OR

[A] Change the order of integration of the following double integral

$$\int_0^{2a} \int_{y^2/4a}^{3a-y} f\,dydx$$

[P.T.O.

[B] Attempt any two

(i) Evalute $\iint_{s}(x^2-y^2)^2\,dx\,dy$

Where S is the region bounded by st line

$x + y = 2, x - y = 3, x + y = 4, x - y = 5$

(ii) Evalute $\iiint_{v}\frac{dx\,dy\,dz}{(x+y+z+1)^3}$

Where V is bounded by planes $x = 0, y = 0, z = 0$ and $x+y+z = 1$

(iii) Find the volume of a sp here $x^2 + y^2 + z^2 = a^2$

3. [A] **Prove that**

(i) grad $(\bar{f}\times\bar{g})=\bar{g}\cdot\text{curl}\,\bar{f}-\bar{f}\cdot\text{curl}\,\bar{g}$

(ii) div $(\phi\bar{f})=\phi\,\text{div}\bar{f}+\bar{f}\cdot(\text{grad}\phi)$

OR

[A] State and prove Green's theorems.

[B] **Attempt any two**

(i) Show that $\nabla\cdot\left(r\nabla\left(\frac{1}{r^3}\right)\right)=\frac{3}{r^4}$ Where $r^2 = x^2 + y^2 + z^2$

(ii) If $\bar{r} = (x, y, z)$, $|\bar{r}| = r$ and $\bar{a} = (a_1, a_2, a_3)$ then prove that

$\nabla\cdot(\bar{a}\times\bar{r}) = 0$ and $\nabla\times(\bar{a}\times\bar{r}) = 2\bar{a}$

(iii) $\oint(3x^2 - 8y^2)dx + (4y-6xy)dy$ where C is curve which is a boundary of the region bounded by $y^2=x$ and $x^2=y$

4. **Attempt any two**

(i) If $r = |\bar{r}|$, $\bar{r} = (x, y, z)$ then prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

(ii) Evaluate $\iint(x^2 + y^2)dx\,dy$ over the region bounded by $x = 1$, $x = 2, y = 1, y = x^2$.

(iii) Find the radious of the curvature of the curve $x^3 + y^3 = 3axy$.

(iv) Evalute $\int_0^\infty\frac{x^k}{k^x}dx$ (where $k > 1$)