P.S.Science and H.D.Patel Arts College, Kadi.

Internal Examination

B.Sc. Semester : V

CC-MATH-502 (Analysis : I)

Dt:24/09/2016 **Time:1:30 to 3.30**

1. (a) If $x, y \in R$ and $x < y$ then prove that there exists $p \in Q$ such that $x < p < y$.

   (b) If $\alpha \in R$, $\beta \in R$ and $\gamma = \{p \in Q / p = r + s, r \in \alpha, s \in \beta\}$ then show that $\gamma \in R$.

   (c) Suppose $\overline{a}$, $\overline{b} \in R^k$ find $\overline{c} \in R^k$ and $r > 0$ such that

   $$|\overline{x} - \overline{a}| = 2|\overline{x} - \overline{b}| \text{ iff } |\overline{x} - \overline{c}| = r.$$

OR

1. (a) State and prove an Archimedean property .

   (b) Let $\alpha \in R$ and $\beta = \{p \in Q / -p - r \notin \alpha \; -p - r \notin \alpha \text{ ,for some } r > 0\}$

   then show that (1) β is cut (2) $\alpha + \beta = 0^*$

   (c) Prove that $\sqrt{12}$ is an irrational.

2. (a) Prove that countable union of countable set is countable.

   (b) Prove that every compact subset of metric space is closed.

   (c) Let R be the set of all real number and $d : R \times R \to R$

   (i) $d(x, y) = 1$ if $x \neq y$ (ii) $d(x, y) = 0$ if $x = y$ then show that

   d is metric on R.

P.T.O.

OR

2. (a) Show that the set F is closed iff its complement is an open set.

(b) Show that a subset E of the real line R^1 is connected if and only if it satisfies the following property.

If $x, y \in E$ and $x < z < y$ then $z \in E$.

(c) Prove that the set Z of all integers is countable.

3. (a) If $E \subset X$ and P is a limit point of E then there exists a sequence $\{P_n\}$ in E such that $P = \lim_{n \to \infty} p_n$

(b) If $\{K_n\}$ is a sequence of compact sets in a metric space X such that $K_n \supset K_{n+1}$ $(n = 1, 2, 3, \ldots)$ and if $\lim_{n\to\infty} diam\ K_n = 0$ then $\bigcap K_n$ consists of exactly one point.

(c) Prove that If $p > 0$ then $\lim_{n \to \infty} \sqrt[n]{p} = 0$.

OR

3. (a) Prove that D'Alembert's ratio test for convergence of series

(b) Suppose (i) the partial sum A_n of $\sum a_n$ from a bounded sequence (ii) $b_0 \geq b_1 \geq b_2 \geq \ldots \geq 0$ (iii) $\lim_{n \to \infty} b_n = 0$

then $\sum a_n b_n$ converges.

(c) Discuss the converges of the series $\sum \frac{z^n}{2^n \cdot n}$