PRAMUKH SWAMI SCIENCE AND H D PATEL ARTS COLLEGE,KADI

B Sc Semester : 5 (MATHEMATICS) : Midterm Exam

PAPER 501 (ABSTRACT ALGEBRA)

[DATE : 23/09/2016 TIME : 2 Hours]

Max Marks : 40

Q-1

(a) Define a Group
Give an example of a non abelian group of order six

(b) State and prove the necessary and sufficient conditions for a non empty subset H of a group G to be a subgroup of G

(c) Let G be a non empty set with binary operation *. and if,

(i) * is associative

(ii) For every $a \in G$, there exists $e \in G$, such that $a*e=a$

(iii) For every $a \in G$, there exists $b \in G$, such that $a*b=e$

Then prove that G is a group under *.

OR

Q-1

(a) In a Group G Prove that

(i) Inverse of an element is unique

(ii) For $a, b \in G$, $(a*b)^{-1} = b^{-1} * a^{-1}$

(b) Suppose $o(a) = n$ for an element a in a group G, then prove that

(i) $O(a^{p}) \leq o(a)$ for $p \in Z$

(ii) $O(a^{p}) = o(a)$ if $(p, n) = 1$

(c) Let $G = R-\{-1\}$, and * be the operation defined on G as $a*b = a+b+ab$ then prove that (G, *) is an abelian group.

Q-2

(a) Prove that the intersection of two subgroups of a group G is always a subgroup of G. Is converse true ? Justify your answer

(b) Show that any two disjoint cycles in S_n are commutative.

(c) For f= (1 2 3)(1 6 5 4 3) $\in S_6$

Find (i) o(f) (ii) Check where f is even or odd.

OR

Q-2

(a) State and prove Lagrange's theorem

(b) Define centre of a group G, Prove that centre of a group G is a subgroup of G

(c) State and prove Euler's theorem

Q-3

(a) State and prove Fermat's theorem

(b) If G = (Z ; +) and H = nZ, then obtain all right cosets of H in G. and find the index of H in G.

(c) Show that a group of prime order is always cycle

OR

Q-3

(a) Prove that the order of a permutation f $\in S_n$ is the least common multiple of the length of its disjoint cycles

(b) For f = (1 3 4 5 6 2) and g = (1 2 4 3)(5 6) $\in S_6$

Find (i) f^3 (ii) $(g^{-1})^2$

(c) Show that the congruent modulo relation is an equivalence relation in a group G

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