P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - I [Mark : 40

24-9-2016] **C. C. Mathematics - III** [2 Hours

1. [A] State and prove Leibnitz's theorem. 4

[B] **Any two** 6

(1) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^m$ then prove that

$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+m^2)y_n = 0$

(2) Prove that $\frac{4}{3} < \frac{\log_e 3}{\cot^{-1} 2} < 4 \quad \forall x \in [1, 3]$

(3) Find approximate value of $\sqrt{10}$ upto 4 decimal places using Taylor's theorem.

2. [A] obtain Reduction formula for $\int_0^{\pi/2} \sin^n x dx$ 4

[B] **Attempt Any two.** 6

(1) Evaluate : $\int_0^1 \frac{x^5 \cos^4(\sin^{-1} x)}{\sqrt{1-x^2}} dx$

(2) Evalute : $\frac{1}{n^3+1} + \frac{4}{n^3+8} + \frac{9}{n^3+27} + \ldots\ldots\ldots\ldots$

(3) Find arc lenght of curve $y = \log(\cos x)$ between

$x = 0$ to $x = \frac{\pi}{4}$

[P.T.O.

3. [A] Prove that line passing through $A(r_1, \theta_1)$ & $B(r_2, \theta_2)$ is 4

$$\frac{\sin(\theta_1 - \theta_2)}{r} = \frac{\sin(\theta - \theta_2)}{r_1} - \frac{\sin(\theta - \theta_1)}{r_2}$$

[B] **Any two.** 6

(1) obtain polur equation of line whose perametric equation are $x = 3 + 7t$, $y = 2 + t$, $t \in R$ also find p and α.

(2) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r = |\bar{r}|$

(3) Prove that $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$ for any vector $\bar{a}, \bar{b}, \bar{c}$

4. **Attempt any three.** 10

(1) obtain redius and centre of circle $x + 2y - 2z + 5 = 0$, $x^2 + y^2 + z^2 - 2y + 2z - 23 = 0$

(2) obtain equation of cone whose vertex is $O(0, 0, 0)$ and guiding curves are $x + y + z = 3$, $x^2 + 2y^2 - z^2 = 2$

(3) obtain equation of cyclider whose axis is $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and guiding curves are $2x^2 + 3y^2 = 1$, $z = 0$

(4) obtain Equation of tangent planes to sphere $x^2 + y^2 + z^2 - 2y - 6z + 5 = 0$ which is parallel to plane $2x + 2y - z = 0$

(5) prove that $yz + zx - xy = 0$ represents right circular cone. also find semi vertical angle and equation of axis.