AAN-467 Seat No. _______

B. Sc. (Sem. V) Examination

October / November – 2016

Mathematics : CC MATH - 503

(Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.

(ii) Figures to the right indicate the marks of the corresponding question.

1 (a) Prove that $\frac{1}{f(D)}\left[e^{ux}\cdot v\right]=e^{ux}\cdot\frac{1}{f(D+u)}\cdot V$. **6**

Where $u=$ costant, $v=$ function of x.

(b) Solve : $\left(D^4-4D^3+8D^2-8D+4\right)y=0$. **6**

(c) Solve : $\left(D^2-5D+6\right)y=\sin 3x$. **6**

OR

1 (a) Prove that **6**

$$\frac{1}{D-\alpha}X=e^{\alpha x}\cdot\int X\cdot e^{-\alpha x}dx$$

Where X is a function of x.

(b) Solve : $\left(D^5-13D^3+26D^2+82D+104\right)y=0$. 6

(c) Solve : $(D-1)^2\left(D^2+1\right)^2 y=\sin^2\frac{x}{2}+e^x$. 6

2 (a) Obtain the first integral of 6

$$x^5\cdot y^{(2)}+3x^3\cdot y^{(1)}+(3-6x)x^2y=x^4+2x-5.$$

(b) Solve : $y^{(2)}=x^2\cdot\sin x$. 6

(c) Solve : $y\,y^{(2)}-\left(y^{(1)}\right)^2-2y^{(1)}=0$. 6

OR

2 (a) Obtain the first integral of 6

$$y^{(2)}+2\tan x\cdot y^{(1)}+3y=\tan^2 x\cdot\sec x.$$

(b) Solve : $y^{(3)}=x\,e^x$. 6

(c) Solve : $2y\,y^{(2)}-3\left(y^{(1)}\right)^2+4y^2=0$. 6

3 (a) Solve : $\cos x\cdot y^{(2)}+\sin x\cdot y^{(1)}-2\cos^3 x\cdot y=2\cos^5 x$. 6

[By changing the independent variable]

(b) Solve : $3x^2 y^{(2)} + \left(2 - 6x^2\right) y^{(1)} - 4y = 0$. **6**

[By factorisation of operator method]

(c) If $y = U(x)$ and $y = V(x)$ are solution of the equation $y^{(2)} + P(x) \cdot y^{(1)} + Q(x) \cdot y = 0$ where $P(x)$ and $Q(x)$ are continuous function of x. Prove that $U \cdot V^{(1)} - V \cdot U^{(1)} = C \cdot e^{-\int P\,dx}$. **6**

OR

3 (a) Solve : $y^{(2)} + (1 - \cot x) y^{(1)} - y \cot x = \sin^2 x$. **6**

(b) Solve : $\left[3x^3 D^2 - 6x^2 D + 2xD - 4\right] y = 0$. **6**

[Using factorisation of operator method]

(c) Solve : $y^{(3)} - 6y^{(2)} + 11 \cdot y^{(1)} - 6y = e^{2x}$. **6**

[By variation of parameter method]

4 Solve any four : **16**

(1) $\left(D^4 + 4\right) y = 0$

(2) $(D - a)^n y = e^{a \cdot x}$

(3) $x^2 y^{(4)} + 1 = 0$

(4) $(1+x)y^{(3)} = y^{(2)}$

(5) $y^{(2)} - 2y^{(1)} + y = x \cdot e^{x} \cdot \log x$

[By variation of parameter method]

(6) $y^{(2)} + 2y^{(1)} + 2y = 1 + x^{2}$

[By method of undetermined cofficients]