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AAN-460

Seat No.________

B. Sc. (Sem. V) Examination

October / November – 2016

Mathematics : CCMATH-502

(Mathematical Analysis-I)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory; there are five question.

(2) Figures to the right indicate marks of the corresponding question.

1 (a) Define the least-upper-bound property of an ordered set. **6**

Prove that : The ordered set $\mathbb{R}$ has the least-upper-bound property.

(b) State and prove : **6**

Schwarz Inequality for a complex number.

(c) Prove that : **6**

If $\bar{x}, \bar{y} \in \mathbb{R}^k$; then $|\bar{x}+\bar{y}|^2 + |\bar{x}-\bar{y}|^2 = 2|\bar{x}|^2 + 2|\bar{y}|^2$

Interpret this geometrically, as a statement about parallelograms.

OR

1 (a) Define a dictionary order on a complex number set C. Is this order turns the set C into an ordered set ? If yes, then this ordered set has the least upper-bound property ? 6

(b) Prove that : The set of rational numbers Q is dense in $\mathbb{R}$. 6

(c) Let $\bar{a}, \bar{b}, \bar{x} \in \mathbb{R}^k$. Then find out $\bar{C} \in \mathbb{R}^k$ and $r > 0$ such that $|\bar{x} - \bar{a}| = 2|\bar{x} - \bar{b}|$ iff $|\bar{x} - \bar{c}| = r$. 6

2 (a) Prove that : If $\{E_n\}_{n=1}^{\infty}$ be a sequence of countable sets, and $S = \bigcup_{n=1}^{\infty} E_n$; then S is countable. 6

(b) Prove that : Compact subsets of metric space are closed. 6

(c) Let (X, d) be a metric space 6

Prove that :

$$\left| d(x, y) - d(z, w) \right| \leq d(x, z) + d(y, w)$$

and

$$\left| d(x, z) - d(y, z) \right| \leq d(x, y); \text{ where } x, y, z, w \in X$$

OR

2 (a) Prove that : **6**

$E \subset \mathbb{R}$ is connected

iff

If $x \in E, y \in E$, and $x < z < y$; then $z \in E$.

(b) Prove that : In $\mathbb{R}^k$, balls are convex. **6**

(c) Prove that : **6**

The set of all rational numbers is countable.

3 (a) Prove that : If $\{p_n\}$ be a sequence in a compact metric space, then there exists subsequence of $\{p_n\}$ converges to a point of X. **6**

(b) Prove that : Compact metric space is complete. **6**

(c) Prove that : $\lim_{n \to \infty} \sqrt[n]{n} = 1$. **6**

OR

3 (a) State the prove : Cauchy criterion for a series. **6**

(b) Prove that : $\sum \frac{1}{np} = \begin{cases} \text{converges} \ ; \text{ if } p > 1 \\ \text{diverges} \ ; \text{ if } p \le 1 \end{cases}$ **6**

(c) Prove that : e is irrational. **6**

4 Prove that : (any **two**) **8**

(a) If monotonic increasing sequence $\{S_n\}_{n=1}^{\infty}$ is bounded, then it converges.

(b) If P is a nonempty perfect set in $\mathbb{R}^k$; then P is uncountable.

(c) Let α be a fix cut and let

$$\beta = \{p \in Q / -p-r \notin \alpha; \text{ for some } r > 0\}$$

Then β is a cut.

5 Attempt any two : 8

(a) If x, y are complex; then $\left||x|-|y|\right| \le |x-y|$.

(b) Give an example of an open cover of the segment (0, 1) which has no finite subcover.

(c) Show that :

Convergence of $\{S_n\}_{n=1}^{\infty}$ implies convergence of $\{|S_n|\}_{n=1}^{\infty}$. Is the converse true ?