AAN-453

AAN-453 Seat No. _______

B. Sc. (Sem. V) Examination

October / November – 2016

Mathematics : CC-MATH-501

(Group Theory)

Time : **3** Hours] [Marks : **70**

Instructions : (1) All questions are compulsory.

(2) The figures to the right indicate the marks of the corresponding question.

1 (a) State and prove the Euler's theorem. **6**

(b) In a commutative group G, the elements a and b are of order m and n respectively. If $(m, n) = 1$ then prove that order of ab is mn. **6**

(c) If H_1 and H_2 are two subgroups of a group G then prove that $H_1 \cup H_2$ is a subgroup of G iff either $H_1 \subset H_2$ or $H_2 \subset H_1$. **6**

OR

1 (a) For a subgroup H of a group G, define a right coset Ha, $a \in G$. Prove that for two element a and b of G, either $Ha = Hb$ or $Ha \cap Hb = \varnothing$. **6**

(b) Show that the set $Z = \{x \in G / xy = yx;\ \text{for each}\ y \in G\}$ is a subgroup of a group G. **6**

(c) Show that a group cannot be a union of its two proper subgroups. **6**

2 (a) Prove that any two disjoint cycles in S_n are commutative. **6**

(b) Prove that $I(G) = \{i_g : g \in G\}$ is a normal subgroup of a (G). **6**

(c) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 6 & 10 & 1 & 8 & 7 & 2 & 5 & 3 & 4 \end{pmatrix} \in S_{10}$ **6**

then express,

(1) f as a composition of a disjoint cycles.

(2) f as a composition of transpositions.

(3) find $O\left(f^1\right)$.

OR

2 (a) Prove that for $n \geq 2$, the set An of even permutations in Sn is a subgroup of order n!/2. **6**

(b) Give an illustration of a non commutative group, each of whose subgroups is normal. **6**

(c) Let $G = R - \{-1\}$ be a group under the b.o. defined by $a * b = a + b + ab$ for $a, b \in G$ and $G' = R_0$ be a group under the b.o. multiplication. Then show that $G \cong G'$. **6**

3 (a) Let $\varphi : (G, *) \to (G', \Delta)$ be a homomorphism with kernel K then show that $G/K \cong \varphi(G)$. **6**

(b) Obtain all subgroups of $(Z_{18}, +_{18})$ and prepare their lattice diagram. **6**

(c) Show that every homomorphic image of an abelian group is abelian. **6**

OR

3 (a) Show that a sub group of a cyclic group is also cyclic. **6**

(b) If $\varphi : (G, *) \to (G', \Delta)$ be a homomorphism then show that for a normal subgroup H' of G', a subgroup $\varphi^{-1}(H')$ of G is a normal subgroup of G **6**

(c) Let $G = \{a, a^2, a^3, a^4, \ldots\ldots, a^{17}, a^{18} = e\}$ be a cyclic group of order 18. Obtain all the generator of G Also obtain all subgroups of G **6**

4 Attempt any two : **8**

(a) Using Fermat's theorem, show that $5^{38} \equiv 4 \pmod{11}$.

(b) If G is a group with $a(G)=\{I_G\}$, then prove that G is commutative.

(c) Let $G=(R,+)$ and $G'=(\{z \in C \mid |z|=1\}, x)$ are groups. Define $\varnothing : G \to G'$ by $\varnothing(x)=e^{ix}$, then show that $\varnothing$ is a homomorphism and find Kernel of $\varnothing$.

5 Attempt any two : **8**

(a) If H and K are two subgroups of a group G and HK = $\{hk / h \in H, k \in K\}$ then prove that HK is a subgroup of G iff HK=KH.

(b) H is a subgroup of group G with index 2 then show that H is a normal subgroup of G

(c) Prove that if $G \neq \{e\}$ is a group having no proper subgroup then G is a cyclic group of prime order.
