AAM-407 Seat No. ______

B. Sc. (Sem. III) Examination

October / November – 2016

CCMATH - 301 : Mathematics

(Calculus & Linear Algebra)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) The figures to the right side indicate the marks of corrosponding question.

1 (a) Let function $\phi(x)$ is continuous at a point $(a,\phi(a))=(a,b)$ and $\lim_{(x,y)\to(a,b)} f(x,y)$ exists and equal to $l\in R$ then prove that $\lim_{x\to a} f(x,\phi(x))$ exists and equal to l. **8**

OR

(a) If $z=f(x,y)$ posseses continuous partial derivative in its domain and if $x=\phi(t)$ and $y=\varphi(t)$ possess continuous derivatives in their domain $[a,b]$ then prove that $\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$. **8**

(b) Attempt any two : **12**

(1) If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$ then prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\cdot\sin\theta$.

(2) If $z = f(u,v)$ and $u = e^x \cos y$, $v = e^x \sin y$ then

prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$.

(3) Using definition of limit of function of two variables, prove that $\lim_{(x,y)\to(1,1)} \frac{x^2 + y^2}{x + y} = 1$.

2 (a) State and prove Euler's theorem. **8**

OR

(a) Explain the Lagrange's method of undetermined multiplies to determine the extreme value of a function of n-variables. **8**

(b) Attempt any two : **12**

(1) If $z = \tan^{-1}\left(\frac{x^{2/3} + x^{1/3} y^{1/3}}{x^5 + y^5}\right)$ then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -\frac{13}{6}\sin 2z \text{ and}$$

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = \frac{13}{6}\sin 2z\left(\frac{13}{3}\cos 2z + 1\right)$$

(2) Find the extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y + 5$.

(3) Find a point within a triangle such that the sum of squares of its distances from the sides of the triagle is minimum.

3 (a) State and prove Rank-Nullity theorem. **8**

OR

(a) Let $S=\left\{\bar{x}_1, \bar{x}_2, \ldots\ldots\ldots, \bar{x}_{m-1}, \bar{x}_m\right\}$ be a finite ordered set in a vector space V with $\bar{x}_1 \neq \theta$ then show that S is linearly dependent if and only if vector $\bar{x}_i$, for some $i, 2 \leq i \leq m$ is a linear combination of precending vector $\bar{x}_1, \bar{x}_2, \ldots\ldots\ldots, \bar{x}_{i-1}$. **8**

(b) Attempt any two : **12**

(i) Find $R(T), N(T), r(T), n(T)$ for linear transformation $T: R^4 \rightarrow R^3$, defined by $T(a,b,c,d)=(a+b, b-d, c-d)$.

(ii) Examine which of the following subsets of vector space R^3 are linearly independent

(1) $\{(1,-2,1),(0,1,2),(1,1,1)\}$

(2) $\{(2,-1,1),(3,2,1),(0,5,1)\}$

(iii) Let linear transformation $T: R^3 \rightarrow R^3$ is defined by $T(x_1, x_2, x_3)=(x_1+x_2+x_3,\ x_2+x_3,\ x_3)$.

Show that T is non-singular and also find T^{-1}.

(iv) Verify the rank-nullity theorem for the linear transformation $T: R^4 \rightarrow R^3$ defined by $T(a,b,c,d)=(a+b, b-d, c-d)$.

4 Attempt any five : **10**

(i) If $u=\sqrt{xy}+\tan^{-1}\frac{y}{x},\ x\neq 0$ then find $x u_x + y u_y$.

(ii) Verify Euler's theorem for the function

$$f(x,y)=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}.$$

(iii) If the function $f(x,y)=\frac{x^3y^3}{\left(x^2+y^2\right)^3},\ (x,y)\neq(0,0)$

$=0, \qquad (x,y)=(0,0)$

then show that f is not differentiable at $(0,0)$.

(iv) If $u=f(r),\ r^2=x^2+y^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{2}{r}f'(r).$$

(v) Let $T:R^2\to R^2,\ T(a,b)=(a+b,\ 2a-b)$. Is T linear transformation ?

(vi) Does the vector $(1,1,2)$ belong to $\left[(1,1,0),(1,0,1),(0,1,1)\right]$ in R^3 ?

(vii) Show that the set $\{(1,0,0),(1,1,0),(1,1,1)\}$ is a basis of R^3.