ACL-1264 Seat No. ________

B. Sc. (Sem. I) Examination

November / December – 2016

Mathematics : Paper-CC-MAT-111

Time : 3 Hours] [Marks : 70

Instructions: (1) All Questions are Compulsory.

(2) The figures to the right indicate marks of the corresponding question.

1 (a) State and Prove: Leibnitz's Theorem [7]

OR

(a) Expand the function f(x)=log(1+x), $-1 < x \leq 1$ in ascending powers of x. [7]

(b) Attempt any two. [8]

(1) If $I_n = \frac{d^n}{dx^n}(x^n \log(x))$ then prove that $I_n = nI_{n-1} + (n-1)!$ and from it deduce that

$$I_n = n!\,[logx + 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \cdots + \tfrac{1}{n}].$$

(2) Using Cauchy mean value theorem prove that $b^b - a^a = c^c$ (blogb-aloga), (0<a<c<b)

(3) Exapand $\tan^{-1} x$ in ascending powers of x.

2(a) Obtain formula for length of arc of continuous curve y=f(x) between lines x=a and x=b. [7]

OR

(a) Obtain reduction formula for $\int sin^m x cos^n x dx$, n∈N and hence deduce reduction formula for $\int_0^{\frac{\pi}{2}} sin^m x cos^n x dx$. [7]

(b) Attempt any two. [8]

(1) Evaluate : $\int_0^1 x^4 (2 - x^2)^{3/2} dx$

(2) Evaluate : $\lim_{n\to\infty}\left[\left(1 + \frac{1^2}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right)^2 \left(1 + \frac{3^2}{n^2}\right)^3 \ldots \left(1 + \frac{n^2}{n^2}\right)^n \right]^{1/n^2}$

(3) Find the volume of sphere of radius a.

3(a) For vectors $\bar{a}, \bar{b}$ and $\bar{c}$ prove that (1) $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

(2) $(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$ [7]

OR

(a) Prove that: Curl $(\Phi\bar{F})$ = (gradΦ)× $\bar{F}$ +Φ(curl$\bar{F}$), where Φ is a scalar and $\bar{F}$ is a vector function. [7]

(b) Attempt any two. [8]

(1) Find reciprocal vector set for the vector set {(1,1,-1), (1,-1,1), (-1,1,1)}

(2) Find divergence and curl of a vector function $\bar{F} = xyz\bar{i} + 3x^2y\bar{j} + (xz^2 - y^2z)\bar{k}$ at point (2,-1,1).

(3) Transform the equation $x^2+y^2=z^2$ into cylindrical and spherical co-ordinates.

4 (a) Derive the equation of a tangent plane of sphere $x^2+y^2+z^2=a^2$ at point (α, β, γ). [7]

OR

(a) Equation of right circular cone having vertex (α, β, γ), axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and semi vertical angle θ in R^3. $(\theta \neq 0, \theta \neq \pi/2)$ [7]

(b) Attempt any two. [8]

(1) Find the value of k, if plane kx + y−2z=9 touches the sphere $x^2+y^2+z^2=9$.

(2) Define Orthogonal Sphere. Prove that two sphere $x^2 + y^2 + z^2 - 2x + 4y - 4 = 0$ and $x^2 + y^2 + z^2 - 6y + 4z8 = 0$ are orthogonal.

(3) Find the equation of right circular cylinder having guiding curve x+y+z=−3, $x^2+y^2+z^2+3x+3y+3z=0$.

5 Attempt any five. [10]

(1) If $y = \frac{x}{x^2-2x-8}, x \neq 2, -4$ then find y_n.

(2) Find the coefficient of x^4 in the expansion of $\log(\cos x)$.

(3) Evaluate : $\int_0^{\pi} sin^3(\frac{\theta}{2})\, d\theta$.

(4) Find the limit of series $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{n+n} + \cdots$

(5) For vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ prove that $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) + (\bar{b} \times \bar{c}) \cdot (\bar{a} \times \bar{d}) + (\bar{c} \times \bar{a}) \cdot (\bar{b} \times \bar{d}) = 0$.

(6) Prove that $\nabla r^2 = 2\bar{r}$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}|$.

(7) Find the equation of sphere whose extremities of diameter are (3,4,0) and (2,3,-1).

(8) Define : Central Conicoids, Elipsoid.
