P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - VI

22-3-2016] **Mathematics - 603 (B)** **[1-30 to 3-00**

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1. **[A]** State & prove Division Algorithm.

OR

[A] The linear Diophantine equation $ax + by = c$ has a solution if and only if d/c where $d = \gcd(a, b)$.

If x_0, y_0 is any particular solution of this equation, then all other solutions are $x = x_0 + \left(\frac{b}{d}\right)t$ & $y = y_0 - \left(\frac{a}{d}\right)t$

t is integer.

[B] **Any two**

(1) Show that the expression $a(a^2 + 2)$ is an integer for all $a \geq 1$

(2) Find gcd (12378, 3054) Also express it as Linear Combination of integers 12378 and 3054

(3) Find positive solution of $172x + 20y = 1000$

2. **[A]** The linear congruence $ax \equiv b \pmod{n}$ has a solution if and only it d/b, where $d = \gcd(a, n)$ If d/b, than it has d mutually incongrent solutions modulo η.

[P.T.O.

OR

[A] Prove that any positive integer N can be written uniquely in terms of powers of b as,

$N = a_0 + a_1 b + a_2 b^2 + ... + a_m b^m$, $b > 1$

[B] **Attempt any two.**

(1) Solve : $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 2 \pmod 7$,

(2) Find the last two digits of the number ${}_9 9^9$

(3) Show that $7 \mid 2222^{5555} + 5555^{2222}$

3. [A] If p is a prime, then $a^p \equiv a \pmod p$ for any integer a.

OR

[A] State and Prove Wilson theorem.

[B] **Attempt any two.**

(1) If p is a prime, prove that

$p \mid a^p + (p-1)!\, a$ amd $p \mid a^p (p-1)! + a$

(2) Express integer '119143' as product of Prime No.

(3) Find Last 3 digit of 7^{999}.
