

**P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI**

**Internal Examination**

**B. Sc. Semester - VI**

21-3-2016]

**Mathematics - 602**

**[1-30 to 3-00**

**1. (a) Attempt any two.**

(i) suppose  $f$  is a continuous mapping of a compact metric space  $X$  into a metric space  $Y$ . then prove that  $f(X)$  is compact.

(ii) Let  $f$  be a monotonic function on  $(a, b)$  then prove that the set of points of  $(a, b)$  at which  $f$  is discontinuous is at most countable.

(iii) Suppose  $f$  is continuous on  $[a, b]$ ,  $f'(x)$  exists at some point  $x \in [a, b]$ ,  $g$  is defined on an interval  $I$  which contains the range of  $f$  and  $g$  is differentiable at  $f(x)$ .

If  $h(t) = g(f(t))$ ,  $a \leq t \leq b$  then prove that  $h$  is differentiable at  $x$  and  $h'(x) = g'(f(x)) \cdot f'(x)$

**(b) Attempt any one.**

(i) suppose  $f$  is continuous for  $x \geq 0$ ,  $f'(x)$  exists for  $x > 0$  and  $f(0) = 0$ ,  $f'$  is monotonically increasing and

$$g(x) = \frac{f(x)}{x}, \quad x > 0$$

then prove that  $g$  is monotonically

increasing.

(ii) If  $f(x) = x + 1, \quad 0 \leq x < 1$   
 $= 2x + 3, \quad 1 \leq x \leq 2$

then discuss the discontinuity of  $f$  at  $x = 1$

**2. (a) Attempt any two.**

(i) If  $f \in R(\alpha)$  and  $g \in R(\alpha)$  on  $[a, b]$  then prove that

$$f + g \in R(\alpha) \text{ on } [a, b] \text{ and } \int_a^b (f + g) d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$$

(ii) State and prove fundamental theorem of calculus.

(iii) If  $f$  is continuous on  $[a, b]$  then prove that  $f \in R(\alpha)$  on  $[a, b]$

(b) Attempt any one.

(i) If  $f$  is bounded on  $[-1, 1]$  and  $f$  is continuous at  $x = 0$

$$\text{If } \beta(x) = 0, \quad x < 0$$

$$= \frac{1}{2}, \quad x = 0$$

$$= 1, \quad x > 0$$

then prove that  $f \in R(\beta)$  on  $[-1, 1]$  and  $\int_{-1}^1 f d\beta = f(0)$

(ii) suppose  $\alpha$  is increasing function on  $[a, b]$   $a \leq x_0 \leq b$ ,  $f(x_0) = 1$  and  $f(x) = 0$  if  $x \neq x_0$  then prove that  $f \in R(\alpha)$

$$\text{on } [a, b] \text{ and } \int_a^b f d\alpha = 0$$

3. (a) Attempt any two.

(i) State and Prove  $M_n$  test for uniform convergence of sequence of function

(ii) If  $K$  is compact metric space If  $f_n \in C(K)$ ,  $n = 1, 2, 3, \dots$  and if  $\{f_n\}$  converges uniformly on  $K$  then prove that  $\{f_n\}$  is equicontinuous on  $K$ .

(iii) suppose  $f_n \rightarrow f$  uniformly on  $E$  in a metric space  $X$ . Let

$x$  be a limit point of  $E$  and suppose  $\lim_{t \rightarrow x} f_n(t) = A_n$ ,  $n = 1, 2, 3, \dots$  then prove that  $\{A_n\}$  converges and

$$\lim_{t \rightarrow x} f(t) = \lim_{n \rightarrow \infty} A_n.$$

(b) Attempt any one.

(i) Show that  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ ,  $0 \leq x \leq 1$  is uniform converges on  $[0, 1]$  and it can integrate term by term.

(ii) If  $f_n(x) = \frac{x^2}{x^2 + (1-nx)^2}$ ,  $0 \leq x \leq 1$ ,  $n = 1, 2, 3, \dots$  then show that  $\{f_n\}$  is uniformly bounded on  $[0, 1]$  but not uniformly convergence.