PRAMUKH SWAMI SCIENCE AND H D PATEL ARTS COLLEGE,KADI

B. Sc. Semester VI (MATHEMATICS)

PAPER : CC 601 (ABSTRACT ALGEBRA)

[DATE : 19/03/2016 **INTERNAL EXAM** **TIME : 1.5 Hours]**

Q-1 (a) Define zero divisor. Prove that a non zero element [m] of a ring $(Z_n +_n \cdot_n)$ is a zero divisor iff m and n are not relatively prime. [6]

OR

Q-1 (a) Show that the characteristics of a ring R with unity is n if and only if n is the smallest positive integer with $(n \bullet 1) = 0$ [6]

Q-1 (b) Attempt Any Two **[8]**

(a) Show that a finite integral domain is a field.

(b) Give an example of a right ideal which is not a left ideal

(c) For a given prime p, prove that $(Z_p, +; .)$ is an integral domain

Q-2 (a) State and prove Eisenstein criterion. [5]

OR

Q-2 (a) In usual notations, for non zero polynomials $f, g \in D[x]$, prove that [fg] = [f] + [g] [5]

Q-2(b) : **Attempt any TWO : [8]**

(a) Show that the polynomial $x^2 + 1$ is irreducible as an element of Q[x] but reducible as an element of $Z_5[x]$.

(b) If the degree of a polynomial $f(x) \in F[x]$ is n, then f(x) has at most n distinct zeros in F.

(c) Suppose f = (0,1,2,1,0,0.....) and g = (1,1,-3,1,0,0,0.......) then find f+g and fg where $f, g \in Z[x]$.

Q-3 (a) Show that the intersection of two ideals is again an ideal in a ring R. what can u say about the Union ? Justify your answer. [5]

OR

Q-3 (a) Let (R; +; .) be a ring with unity. Show that the mapping

$\phi : (Z; +; .) \rightarrow (R; +; .)$ where $\phi(n) = n1$, $n \in Z$ is a

homomorphism with $K_\phi = \langle m \rangle$ if the characteristic of ring R is m.

[5]

Q-3 (b) Attempt any Two [8]

(a) For Ideals I_1 and I_2 of a ring R, show that $I_1 + I_2$ is also an ideal.

(b) Suppose R is a ring with unity. If, homomorphism

$\emptyset : (R; +; .) \rightarrow (R'; \oplus; \odot)$ with $\emptyset(1) \neq 0$, then show that $\emptyset(1)$ is a unity element of ring $\emptyset(R)$.

(c) If $\emptyset : (R; +; .) \rightarrow (R'; \oplus; \odot)$ is a homomorphism and U is an ideal of R then show that $\emptyset(U)$ is an ideal of $\emptyset(R)$.

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