

**P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI**

**Internal Examination**

**B. Sc. Semester - IV**

**18-3-2016]**

**Mathematics - 401**

**[1-30 to 3-00**

**(Advanced Calculus)**

1. [A] Obtain the radius of curvature of a curve  $y = f(x)$

**OR**

- [A] Obtain the radius of curvature of curve defined in  $P - \Psi$  equation say  $p = f(\Psi)$ .

[B] Attempt any two

- (i) Find the radius of curve  $x^{2/3} + y^{2/3} = a^{2/3}$   
(ii) Find the radius of curvature of curve  $r = a(1 - \cos\theta)$   
(iii) Find the centre of curvature of curve

$$x = a \left( \cos\theta + \log \tan \frac{\theta}{2} \right), y = a \sin\theta.$$

prove that its evolute is catenary

$$y = \frac{a}{2} \left( e^{x/a} + e^{-x/a} \right)$$

2. [A] Prove

$$(i) \beta(m, n) = \int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta.$$

$$(ii) \beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} \, dx$$

**OR**

- [A] Obtain the relation between Beta and Gamma function.

**[B] Attempt any two**

(i) Evaluate  $\int_0^1 \sqrt{x} \sqrt[3]{1-x^2} dx$

(ii) Change the order of integration  $\int_0^{2a} \int_{y^2/4a}^{3a-y} f dy dx$

(iii) Evaluate  $\iint_S (x^2 - y^2)^2 dx dy$

Where S is the region bounded by st line  
 $x + y = 2, x - y = 3, x + y = 4, x - y = 5$

3. [A] State and prove Green's theorem.

**OR**

[A] Prove that

$$\text{grad} (\bar{f} \cdot \bar{g}) = \bar{f} \times \text{curl} \bar{g} + \bar{g} \times \text{curl} \bar{f} + \bar{f} (\nabla \cdot \bar{g}) + \bar{g} (\nabla \cdot \bar{f})$$

**[B] Attempt any two**

(1) Show that  $\nabla \cdot \left( r \nabla \left( \frac{1}{r^3} \right) \right) = \frac{3}{r^4}$  Where  $r^2 = x^2 + y^2 + z^2$

(2) Compute  $\oint (y^2 dx - x^2 dy)$  over the triangle whose vertices are (1, 0), (0, 1), (-1, 0)

(3) Verify Green's theorem

$\oint (x + y) dx + (x - y) dy$  Where C is circumference of the circle  $x^2 + (y - 1)^2 = 1$