HG-225-226-227 Seat No.______

B. Sc. (Sem. VI) Examination

March / April – 2015

Mathematics

CC-MATH-604-A : Graph Theory

CC-MATH-604-B : Mechanics - II

CC-MATH-604-C : Operations Research - II

(New Course)

Time : **3** Hours] [Total Marks : **70**

Mathematics

CC-MATH-604-A : Graph Theory

Instructions : (1) All questions are compulsory, there are five questions.
(2) Figures to the right indicate marks of the corresponding question.

1 (a) Define Union of graph with illustration. **6**

OR

(a) Define intersection of graph with illustration. **6**

(b) Attempt any two : **12**

(i) Define complete graph with illustration.

(ii) Define incidence and Degree of a vertex with illustration.

(iii) Define eccentricity of a vertex with illustration.

2 (a) Define Cut set with illustration. **6**

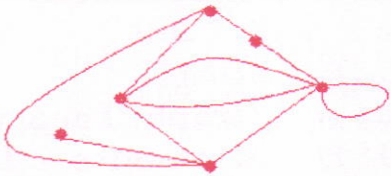
OR

(a) Define Vertex connectivity of graph with illustration. **6**

(b) Attempt any **two** : 12

(i) Define Separable graph with illustration.

(ii) Check planarity of the given graph below.



(iii) Define a Cut set matrix and state necessary and sufficient condition for combinatorial dual.

3 (a) Explain properties incidence matrix. 6

OR

(a) Explain adjacency matrix with illustration. 6

(b) Attempt any **two** : 12

(i) Define covering with illustration.

(ii) Explain diacyclization with illustration.

(iii) Define Chromatic partition with illustration.

4 Attempt any **four** : 16

(1) Defines :

(i) Circuit sub space

(ii) Cut set subspace.

(2) Define branch with illustration.

(3) Define Chord with illustration.

(4) Prove that a vertex V of a tree is cut vertex if $d(V)>1$.

(5) Define Binary Tree with illustration.

Mathematics
CC-MATH-604-B : Mechanics - II

1 Attempt any **three** : **24**

(a) A particle is moving along the path $r = ae^{\theta}$ in such a way so that the radical component of its acceleration is always zero. Prove that $\frac{d\theta}{dt}$ is constant.

(b) A particle is projected with velocity V and making an angle θ with horizontal. Find the expression for the horizontal range and the time of flight.

(c) For a rigid body moving parallel to a fixed plane. In usual notations prove that $T = \frac{1}{2}mv^2 + \frac{1}{2}IW^2$.

(d) Moving along the x-axis there are two partical with $x = 10 + 6t;\ x = 3 + t^2$. Then find out velocity at the time of encounter of each other.

2 Attempt any **two** : **16**

(a) Define a simple harmonic motion and obtain its equation in the form

$$x = a\left(1 - 2\sin^2\left(\frac{1}{2}\sqrt{\mu}\,t\right)\right).$$

(b) Define a compound pendulum and in usual notation prove that its periodic time T is obtain by the following equation :

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\alpha + \frac{K_0^2}{\alpha}}$$

(c) A particle describes the central orbit $r^2 = a^2\left(2\cos^2\theta - 1\right)$; the centre of force being a pole. Show that the law of force varies inversely as r^7.

3 Attempt any **two** : **16**

(a) A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution is e; then prove that their velocities after impact is in the ratio $\frac{1-e}{1+e}$

(b) Two smooth sphere of mass m_1 and m_2 moving with velocity u and v in the opposite direction, collide in the straight line. If sphere of mass m is brought to rest by the impact; then show that $um_1(1+e) = v\left(m_2 - em_1\right)$.

(c) State and prove : The theorem of parallel axes for the moment of inertia of rigid body.

4 Attempt any **two** : **14**

(a) A particle is moving under a central force $bu^2 + cu^4$ (where b and c are constants) in a circular orbit of radius a. Find the condition for the circular orbit to be stable.

(b) If polar equation of central orbit is $r = a \cdot \sqrt[n]{2\cos^2\left(\frac{n\theta}{2}\right) - 1}$; then find central force.

(c) A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 15 cm, then find out the rate at which the thickness of ice decreases.

Mathematics

CC-MATH-604-C : Operations Research - II
(New Course)

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate marks of the question.

1 (a) Prove that transportation problem has a triangular basis. **10**

(b) Solve the following assignment problem : **10**

Job / Man	Cost matrix				
	1	2	3	4	5
a	3	2	7	4	8
b	5	4	3	8	5
c	3	7	9	1	2
d	4	2	6	5	7
e	2	8	4	6	6

OR

1 (a) Explain Hungarian method of solving assignment problem. **10**

(b) Solve the following transportation problem for optimal cost : **10**

To / From	D_1	D_2	D_3	a_i
O_1	4	8	8	75
O_2	16	24	16	80
O_3	8	16	24	70
b_j	40	80	55	

2 (a) Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing time (in hours) are given in the following table : **10**

Job :	1	2	3	4	5
Machine A :	8	10	6	7	11
Machine B :	5	6	2	3	4
Machine C :	4	9	8	6	5

(b) There are 4 jobs each of which has to go through the machines $M_i, (i = 1, 2, 3, ..., 6)$ in order $M_1, M_2, ..., M_6$. Processing times are given : **10**

Jobs	A	B	C	D
M_1	20	19	13	22
M_2	10	8	7	6
M_3	9	11	10	5
M_4	4	8	7	6
M_5	12	10	9	10
M_6	27	21	17	14

Determine a sequence of these four jobs which minimizes the elapsed time T.

OR

2 (a) Describe Johnson's Algorithm for n-jobs 2-machine. **10**

(b) Following table shows the machine time (in hours) for 5 jobs to be processed on two different machine : **10**

Job :	1	2	3	4	5	6
Machine A :	3	9	4	7	8	6
Machine B :	6	4	8	3	9	5

Passing is not allowed. Find the optimal sequence in which job should be processed.

3 (a) For a real valued function $f(x, y)$, both **10**

$\max_x \min_y f(x, y)$ and $\min_y \max_x f(x, y)$ exists.

Prove that $\max_x \min_y f(x, y) \le \min_y \max_x f(x, y)$.

(b) Solve the following game problem by converting it to LPP : **10**

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 4 & 3 \\ -2 & 4 \\ 1 & 5 \end{bmatrix} \end{array}$$

OR

3 (a) Define : **10**

(i) Game

(ii) Zero sum game

(iii) Saddle point

(iv) Value of the game

(v) Mixed strategy.

(b) Solve the graphical solution of **10**

$$\begin{array}{c} \quad B \\ A\begin{bmatrix} 2 & -1 \\ 5 & 2 \\ 1 & 3 \\ -2 & 1 \end{bmatrix} \end{array}$$

4 Attempt any **two** : **10**

(1) Find the saddle point of the game :

$$\begin{array}{cc} & \text{Player } B \\ & \begin{matrix} b_1 & b_2 & b_3 \end{matrix} \\ \text{Player } A \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{bmatrix} -3 & -2 & 6 \\ 2 & 1 & 2 \\ 5 & -2 & -4 \end{bmatrix} \end{array}$$

(2) There are five jobs, each of which must go through the two machines A and B in the order AB. Processing times are given below :

Job :	1	2	3	4	5
Time for A :	5	1	9	3	10
Time for B :	2	6	7	8	4

Determine a sequence for five jobs that will minimize the elapsed time T.

(3) Find the value of the 2×2 game when saddle point does not exist.

$$\begin{array}{c} \quad B \\ A\begin{bmatrix} 5 & 2 \\ 3 & 8 \end{bmatrix} \end{array}$$