HG-217-218 Seat No._____

B. Sc. (Sem. VI) Examination

March / April – 2015

Mathematics : Paper - 603

CC-MATH-603-A : General Topology

CC-MATH-603-B : Number Theory

Time : 3 Hours] [Total Marks : **70**

Mathematics : Paper - 603

CC-MATH-603-A : General Topology

Instruction : All questions are compulsory, there are five questions.

1 (a) Define topological space.

If

$T = \{A \cup R \mid A = R \text{ or } A = \phi \text{ or } A - (a, \infty), a \in R\}$; then prove that (R, T) is a topological space.

(b) Define the closure of a subset of a topological space. Prove that : A is closed iff $\overline{A} = A$; where A is a subset of a topological space.

(c) Prove that :

A subset O of topological space is open

iff

O is a neighbourhood of each of its points.

OR

1 (a) Define neighbourhood of a point in a topological space.

If $T = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$

be a topology on $X = \{a, b, c, d, e\}$; then find out

all neighbourhoods of e

(b) Let A and F are subsets of a topological space X s.t. F is closed and $A \subset F$. Then prove that $\bar{A} \subset F$.

(c) Prove that : In a topological space;

$\bar{A}$ is the smallest closed set containing A;

Where A is a subset of a topological space.

2 (a) Define interior of a subset of a topological space.

If A be a subset of a topological space; then prove that $Int.(A) = \bigcup_{\alpha \in I} O_\alpha$; where $\{O_\alpha\}_{\alpha \in I}$ is the family of all open sets contained in A.

(b) Define continuity of a function $f:(X,T)\to(Y,T')$ at a point $p\in X$.

Let $T=\{X,\phi,\{a\},\{b\},\{a,b\},\{b,c,d\}\}$ be a topology on $X=\{a,b,c,d\}$.

Define $f:X\to X$ by $f(a)=b$, $f(b)=d$, $f(c)=b$, $f(d)=c$

(i) Is f is continuous at c ?

(ii) Is f is continuous at d ?

(c) Prove that :

$f:(X,T)\to(Y,T')$ is continuous

iff

for each subset A of X, $f(\overline{A})\subset\overline{f(A)}$.

OR

2 (a) Define homeomorphic topological space.

Prove that : R and $(-1,1)$ are homeomorphic topological spaces.

(b) Prove that :

A function $f:(X,T)\to(Y,T')$ is continuous

iff

for each open subset O of Y; $f^{-1}(O)$ is an open subset of X.

(c) Prove that :

A necessary and sufficient condition that two topological spaces (X, T) and (Y, T') be homeomorphic is that there is a function $f : X \to Y$ such that

(i) f is injective

(ii) f is surjective

(iii) $O \in T$ iff $f(O) \in T'$

3 (a) Define subspace of a topological space.

Let Y be a subspace of a topological space X and let $p \in Y$.

Then prove that :

a subset N' of Y is a relative neighbourhood of p iff

$N' = N \cap Y$; where N is a neighbourhood of p in X.

(b) Define topological property :

Show that :

(i) Length is not a topological property.

(ii) Boundness is not a topological property.

(c) State and prove : Intermediate - Value theorem.

OR

3 (a) Define connected subset of a topological space.

Let A be a connected subset of a topological space X and let $A \subset B \subset \bar{A}$. Then prove that : B is also connected.

(b) Prove that : Connectedness is a topological property.

(c) Prove that :

In a topological space X ; let $b \in C_{mp}(a)$

then $C_{mp}(b) = C_{mp}(a)$.

4 Attempt any **two** :

(a) Let A be a subset of a topological space and a point $x \notin \overline{A}$. Then show that $x \notin F$, for some closed set F containing A.

(b) Let A be a subset of a topological.

Prove that :

A is closed iff Bdry $(A) \subset A$.

(c) Prove that :
In a topological space, component is a closed set.

5 Attempt any **two** :

(a) Suppose that $X \neq \phi$; (Y, T') be a topological space and $f : X \to Y$ be a function and $T = \left\{ f^{-1}(G) \mid G \in T' \right\}$. Show that ; T is a topology on X.

(b) Let a function $f : X \to Y$ be given.

Prove that $f : \left(X, 2^X\right) \to (Y, T')$ is always continuous; as is $f : (X, T) \to \left(Y, \{\phi, Y\}\right)$; where T' is any topology on Y and T is any topology on X.

(c) Let A and B be subsets of a topological space X. If A is connected, B is open and closed, and $A \cap B \neq \phi$. Prove that $A \subset B$.

Mathematics : Paper - 603
CC-MATH-603-B : Number Theory

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate marks of the question.

1 (a) The linear Diophantine equation $ax+by=c$ has a solution if and only if d/c where $d=\gcd(a,b)$. Prove if x_0, y_0 is a solution of this equation, then all other solutions are given by $x=x_0+\left(\frac{b}{d}\right)t$, $y=y_0-\left(\frac{a}{d}\right)t$, where t is any integer. **8**

OR

(a) Prove Binomial theorem by induction **8**

$$(a+b)^n=\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+....+\binom{n}{n}b^n$$

(b) Attempt any two of the following : **12**

(1) Show that the square of any integer is either of the form $3k$ or $3k+1$.

(2) Solve the Diophantine equation $56x+72y=40$.

(3) If a and b are integers not both of them are zero and $d=gcd(a,b)$ then $d=ax+by$.

2 (a) Define linear congruence relation. If a, b, c are integers, $c > 0$ and $ac \equiv bc \pmod{n}$ then $a \equiv b \pmod{n/d}$, where $d = \gcd(c, n)$. 8

OR

(a) Let $n > 0$ be fixed and a, b, c be any integers 8

(i) if $a \equiv b \pmod{n} \Rightarrow b \equiv a \pmod{n}$

(ii) if $a \equiv b \pmod{n}, b \equiv c \pmod{n} \Rightarrow a \equiv c \pmod{n}$

(b) Attempt any two of the following : 12

(1) Prove that $1^5 + 2^5 + 3^5 + \ldots\ldots + 100^5$ is divisible by 4.

(2) Solve the linear congruence equation $9x \equiv 21 \pmod{30}$

(3) If p_n is the n^{th} prime number, then $p_n \leq 2^{2^{n-1}}$ by mathematical induction.

3 (a) State and prove Euler's theorem. 8

OR

(a) If p is a prime and $p \nmid a$, then 8

$a^{p-1} \equiv 1 \pmod{p}$.

(b) Attempt any two of the following : 12

(1) If p and q are distinct primes such that $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then $a^{pq} \equiv a \pmod{pq}$.

(2) Find the last two digit of 3^{256} by means of Euler's theorem.

(3) If $n > 2$ then $\phi(n)$ is even.

4 Answer the following questions : 10

(1) If a/c and b/c, with $\gcd(a, b) = 1$, then ab/c.

(2) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$

(3) Find the remainder when $1! + 2! + 3! + \ldots\ldots + 100!$ is divisible by 12.

(4) Verify $\phi(m \cdot n) = \phi(m) \cdot \phi(n)$ holds when $m = 36$, $n = 10$.

(5) If n and $n+2$ are pair of twin primes then $\phi(n+2) = \phi(n) + 2$.