



HG-210

Seat No. _____

B. Sc. (Sem. VI) Examination

March / April - 2015

CC-MATH-602 : Mathematics

(Mathematical Analysis - II)

Time : 3 Hours]

[Total Marks : 70

- Instructions :** (1) All questions are compulsory.
(2) There are total five questions.
(3) Figures to the right side indicate emarks of corresponding question.

- 1 (a) Suppose f is a real differentiable function on $[a, b]$ and $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$. **6**
- (b) Suppose X and Y are metric spaces and $E \subset X$, p is a limit point of E and $f: E \rightarrow Y$ be a continuous, then prove that $\lim_{x \rightarrow p} f(x) = q$ if and only if $\lim_{n \rightarrow \infty} f(p_n) = q$ for every sequence $\{p_n\}$ in E such that $p_n \neq p$ and $\lim_{n \rightarrow \infty} p_n = p$. **6**

- (c) State the type of discontinuity for following functions : 6

(i) $f(x) = x - [x]$ at $x = 2$

(ii) $f(x) = \begin{cases} x+1 & 0 \leq x \leq 1 \\ 2x+3 & 1 \leq x \leq 2 \end{cases}$ at $x = 1$.

OR

- 1 (a) If real function f and g are continuous on 6
 $[a, b]$ and differentiable in (a, b) . Then prove
that there exist $x \in (a, b)$ such that
$$[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$$
- (b) Show that continuous image of connected set 6
is connected.
- (c) Suppose $a \in R'$. f is twice differentiable real 6
function on (a, ∞) and M_0, M_1, M_2 are l.u.b.
of $|f(x)|$, $|f'(x)|$ and $|f''(x)|$ respectively on
 (a, ∞) then prove that $M_1^2 \leq 4M_0M_2$.

- 2 (a) If $f \in R[a, b]$ and F is differentiable function on $[a, b]$ such that $F'(x) = f(x)$ for $\forall x \in [a, b]$ 6

then prove that $\int_a^b f(x) dx = F(b) - F(a)$.

- (b) If $\bar{F}: [a, b] \rightarrow R^K$ and $\bar{F} \in R(\alpha)$ where α is monotonically increasing then prove that 6

(i) $|\bar{F}| \in R(\alpha)$

(ii) $\left| \int_a^b \bar{F} d\alpha \right| \leq \int_a^b |\bar{F}| d\alpha$

- (c) Let f be a continuous function on $[0, 2]$ and $\alpha: [0, 2] \rightarrow R$ is defined by 6

$$\begin{aligned} \alpha(x) &= 0 & 0 \leq x < 1 \\ &= c & x = 1 \\ &= 1 & 1 \leq x \leq 2 \end{aligned}$$

Then prove that $f \in R(\alpha)$ on $[0, 2]$.

OR

- 2 (a) If F is continuous on $[a, b]$ then prove that 6
 $f \in R(\alpha) [a, b]$.

- (b) If F and G are differentiable on $[a, b]$ and 6
 $F' = f \in R[a, b]$ $G' = g \in R[a, b]$ then prove that

$$\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$$

- (c) If f is defined on $[0, 1]$ 6

$$f(x) = \frac{1}{2^n} \quad \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n} \quad n = 0, 1, 2, \dots$$

$$f(x) = 0 \quad \alpha(x) = x \quad \text{for } x \in [0, 1]$$

Then prove that $f \in R(\alpha)[0, 1]$ and

$$\int_0^1 f(x)dx = \frac{2}{3}$$

- 3 (a) Prove that $C(X)$ is complete metric space 6
where X is metric space.

(b) Suppose $f_n \rightarrow f$ uniformly on set E . Let x be 6

a limit point of E and $\lim_{t \rightarrow x} f_n(t) = A_n$

for $n = 1, 2, \dots$ then prove that $\{A_n\}$ converges

and $\lim_{t \rightarrow x} \lim_{n \rightarrow \infty} f_n(t) = \lim_{n \rightarrow \infty} \lim_{t \rightarrow x} f_n(t)$

(c) If $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in R$, then prove that $\{f_n\}$ 6

does not converge to f uniformly.

OR

3 (a) Let K be a compact metric space and $f_n \in C(K)$ 6

for $n = 1, 2, 3, \dots$. If the sequence $\{f_n\}$

converges uniformly on K then prove that

$\{f_n\}$ is equicontinuous on K .

(b) Let α be monotonically increasing on 6

$[a, b]$ and suppose $f_n \in R(\alpha)[a, b]$

$n = 1, 2, \dots$, $f_n \rightarrow f$ uniformly on $[a, b]$

then prove that $f \in R(\alpha)[a, b]$ and

$$\int_a^b f d\alpha = \lim_{n \rightarrow \infty} \int_a^b f_n d\alpha.$$

(c) Prove that

6

$$\cos x + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{2^3} + \frac{\cos 4x}{2^4} + \dots$$

uniformly converges on R .

4 Attempt any **two** :

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(a) Let f be monotonic function on (a, b) then prove that set of points of (a, b) at which f is discontinuous is at most countable.

(b) If f is monotonic function on $[a, b]$ and α is continuous function on $[a, b]$ then prove that $f \in R(\alpha) [a, b]$.

(c) The sequence of function $\{f_n\}$ converges uniformly on set E if and only if $\forall \epsilon > 0 \exists$ a integer N such that $|f_n(x) - f_m(x)| \leq \epsilon$, where $m \geq N, n \leq N, x \in E$.

- (a) Explain that L'Hospital rule does not hold true for complex valued function.
- (b) By using definition of R-S integral prove that

$$\int_0^2 x^2 d(2x) = \frac{16}{3}$$

- (c) If $f_n \in R(\alpha)$ on $[a, b]$ and if $f(x) = \sum_{n=1}^{\infty} f_n(x)$

the series converges uniformly on $[a, b]$ then

prove that
$$\int_a^b f d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n d\alpha.$$
