P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - V

7-10-2015] **C.C. Maths - 502** **[1-30 to 3-00**

Mathematical Analysis-I

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1. **[A]** State and prove Archimedean property.

   [B] If $\alpha \in R$, $\beta \in R$ then show that $\alpha + \beta = \{s + t / s \in r, t \in \beta\}$ is also cut.

   [C] Suppose $\bar{a}, \bar{b} \in R^K$, find $\bar{c} \in R^K$ and $r > 0$ such that $|\bar{x} - \bar{a}| = 2|\bar{x} - \bar{b}|$ iff $|\bar{x} - \bar{c}| = r$

OR

1. **[A]** Show that an ordered set R has the least upper bound propety.

   [B] If $x, y \in R$ and $x < y$ then there exists $P \in Q$ such that $x < p < y$.

   [C] Let A be a non-empty set of read numbers which is bounded below. Let B be the set of all numbers -x, where $x \in A$. then prove that inf A = -sup(-B)

2. **[A]** Show that a set F is closed if and only if its complement is an open set.

   [B] Prove that compact subset of metric space are closed.

   [C] Let R be the set of all real numbers and $d : R \times R \to R$,
   $d(x, y) = 1$, if $x \neq y$
   $= 0$, if $x = y$
   then show that d is metric on R.

[P.T.O.

OR

2. **[A]** Show that a subset E of the real line R' in connected if and only if it has the following properly.
If $x, y \in E$ and $x < z < y$ then $z \in E$

[B] Let $\{E_n / n = 1, 2, 3, ...\}$ be a sequence of countable sets and $S = \bigcup_{n=1}^{\infty} E_n$ then show that S is countable set.

[C] Show that R is not compact.

3. **[A]** If X is a compact metric space and $\{P_n\}$ is a caugchy sequence in X then prove that $\{P_n\}$ converges to some point of X.

[B] If $\bar{x}_n = (x_{1n}, x_{2n}, ... x_{kn}) \in R^k$, $n = 1, 2, 3,...$ then show that the sequence $\{\bar{x}_n\}$ converges to $\bar{x} = (x_{1n}, x_{2n}, ... x_{kn})$ if and only if $\lim_{n\to\infty} x_{jn} = x_j$, $(1 \le j \le k$

[C] Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$

OR

3. **[A]** State and prove Ratio test.

[B] Suppose (i) The partial sum A_n of Ian from a bounded sequence (ii) $b_o \ge b_1 \ge b_2 \ge ..... \ge 0$ (iii) $\lim_{n\to\infty} b_n = 0$ then $\Sigma a_n b_n$ converges

[C] If $a_n > 0$ and suppose Σa_n diverges then show that $\sum \frac{a_n}{a_{n+1}}$ diverges.
