PRAMUKH SWAMI SCIENCE AND H D PATEL ARTS COLLEGE,KADI

B. Sc. Semester V (MATHEMATICS)

PAPER : CC 501 (Group Theory)

[DATE : 06/10/2015 INTERNAL EXAM TIME : 1.5 Hours]

Q-1 (a) Define a group. In a group G, prove that

(1) Identity element is unique

(2) Inverse of an element is unique [6]

OR

Q-1 (a) Suppose o(a) = n for an element a in a group G.Then prove that

(1) $o(a^P) \le o(a)$ (2) $o(a) = o(bab^{-1})$ for $b \in G$ [6]

Q-1 (b) Attempt Any Two **[8]**

(a) If G is a finite group of even order the show that there exists at least one element a ≠ e in G such $a = a^{-1}$.

(b) If $a \in G$ is of order n, then prove that $a^m = e$ for some integer m if and only if n/m.

(c) Show that a group cannot be the union of its two proper subgroups.

(d) Show that a group of prime order is cyclic.

Q-2 (a) State and prove Lagranges theorem . [5]

OR

(a) If n is a prime number then prove that $a^n \equiv a \pmod{n}$ for any integer n. [5]

Q-2(b) : Attempt any TWO : **[8]**

(a) For $f \in S_{12}$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 6 & 1 & 9 & 12 & 4 & 11 & 10 & 8 & 3 & 5 & 2 & 7 \end{pmatrix}$$

Find (i) $\mathbf{f}^2$ (ii) f^{-1} (iii) o(**f**) and (iv) check where f is odd or even permutation

(b) Using the Euler's theorem find the remainder obtained on dividing 3^{256} by 14.

(c) If G = (Z, +) and H = (nZ, +) then obtain all right costs of H in G and also find the index of H in G..

(d) Any two disjoint cycles in S_n are commutative.

Q-3 (a) Suppose group G is Isomorphic to group G′ then prove that G is commutative if and only if G′ is commutative. [5]

OR

(a) A subgroup H of a group G is a normal subgroup of G if and only if aHa^{-1} is a subset of H . [5]

Q-3 (b) Attempt any Two **[8]**

(a) If the quotient group G/Z is cyclic then prove that G is a commutative group, where Z is the centre of group G.

(b) Prove that a subgroup of index 2 in a group is a normal subgroup.

Is alternating subgroup A_n of symmetric group S_n a normal ? Justify your answer.

(c) Show that $(Z_5 - \{[0]\}, *_5)$ is a group. Solve $[4] *_5 Y = [2]$

(d) Show that the n^{th} root of unity form an abelian group under multiplication.

(e) In usual notations show that $I(G) = \{ i_g : g \in G\}$ is a normal subgroup of a(G).

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