

P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - III

5-10-2015]

C. C. Mathematics - 301

[1-30 to 3-00

Calculus and Linear Algebra

1. [A] Define Continuity

Discuss the continuity of the function

$$f(x, y) = \begin{cases} \tan^{-1} \left(\frac{y}{x} \right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

[B] Attempt any two

(i) Evaluate f_x and f_y for the function

$$f(x, y) = \begin{cases} \frac{x^2 + y^2}{x + y} & , x + y \neq 0 \\ 0 & , x + y = 0 \end{cases}$$

(ii) Find $f_{xy}(0, 0)$ for the function

$$f(x, y) = \begin{cases} \frac{xy(x^2 + y^2)}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

(iii) If $u = x \log x + y \log y$ then prove that $u_{xy} = u_{yx}$.

2. [A] Define Homogeneous function If $u = \cos^{-1} \frac{x^2 + y^2}{x + y}$,

$$x + y \neq 0 \text{ then P.T. } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\cot x$$

[B] Attempt any two

(i) if $u = \tan^{-1} \left(\frac{x^2 + y^2}{x - y} \right)$ then Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

(ii) Find the Talor's expansion of $e^{ax} \cos y$ by about $(0, 0)$ upto and including the terms of 2nd degree.

(iii) Expand $x^3 + xy^2$ in power of $(x - 2)$ and $(y - 1)$

3. **[A]** Define vector Space P.T. $(\mathbb{R}^+, +, \cdot)$ is a vector space where $+: \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$

$$\text{for } x, y \in \mathbb{R}^+ \quad x + y = xy$$

$$\cdot: \mathbb{F} \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$$

$$\text{for } a \in \mathbb{F}, x \in \mathbb{R}^+ \quad a \cdot x = x^a$$

[B] Attempt any two

(i) Check the following subsets of \mathbb{R}^3 os Subspce or not:

$$u = \{(a_1, a_2, a_3) / a_1 - 2a_2 + a_3 = 0\}$$

(ii) If $e_1 = (3, 2, 1)$, $e_2 = (2, 1, 0)$, $e_3 = (1, 0, 0)$ then express the vector $(3, 5, 2) \in \mathbb{R}^3$ as a linear combination of

$$e_1, e_2, e_3$$

(iii) Prove that the Set $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ form a basis for \mathbb{R}^3 .