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HBY-1012

Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2015

CC-MATH - 402 : Mathematics

(Advanced Linear Algebra)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) There are four questions and all questions are compulsory.
(2) Figures in the right side indicate the marks of question.

1 (a) Prove that **8**

A square matrix A is invertible if and only if the corresponding linear transformation T is non-singular.

OR

(a) If A and B are $n \times$ n invertible matrices then prove that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$. **8**

(b) Attempt any two : **12**

(1) Solve the system of equations :

$$x - y + z = 0$$

$$2x + y - 3z = 0$$

$$-x + y + 2z = -1$$

(2) Determine the rank of matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \end{bmatrix}$$

by row reduction method.

(3) If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ and

$B_1=\{(1,1,1),(1,0,0),(0,1,0)\}$

$B_2=\{(1,2,3),(1,-1,1),(2,1,1)\}$

are ordered basis of vector space R^3 then find a linear transformation $T:R^3\to R^3$ such that $A=[T:B_1,B_2]$.

2 (a) Define Bilinear form which of the following f defined R^2 are bilinear form ? 8

(i) $f(\overline{x},\overline{y})=x_1y_2-x_2y_1$

(ii) $f(\overline{x},\overline{y})=(x_1-y_1)+x_2y_2$

where $\forall\,\overline{x}=(x_1,x_2)$ and $\overline{y}=(y_1,y_2)\in R^2$.

OR

(a) Let V be a n-dimensional vector space and $B=\{x_1,x_2,\ldots\ldots\ldots x_n\}$ be an ordered basis of V then for any ordered set $S=\{a_1,a_2,\ldots\ldots,a_n\}$ of n-scalars $\exists$ a unique linear functional f on V such that $f(x_i)=a_i$ where $i=1,2,\ldots\ldots\ldots,n$. 8

(b) Attempt any **two** : **12**

(1) Find dual basis from the basis $\{(1,-1,3),(0,1,-1),(0,3,-2)\}$.

(2) If ϕ and ψ are two bilinear form on vector space $v(k)$ and $f: v \times v \to k$ defined $f(u,v) = \phi(u) \cdot \psi(v)$, then show that f is a bilinear form on v.

(3) By using Gramm Schmitz's process obtain the orthonormal basis from the basis $\{(-1,1,1),(1,-1,1),(1,1,-1)\}$.

3 (a) State and prove Cayley-Hamilton theorem. **8**

OR

(a) Prove that eigen vector of symmetric linear map corresponding to different eigen values are perpendicular to each other.

(b) Attempt any **two** : **12**

(1) Find the minimal polynomial for matrix

$$A = \begin{bmatrix} -4 & -4 & 5 \\ 7 & 4 & -1 \\ 5 & 7 & -4 \end{bmatrix}$$

(2) Find eigen values and corresponding eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(3) Find characteristic equation of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

4 Attempt any **three** : **10**

(1) State and prove Schwartz's inequality.

(2) Define :

(i) Inner product space

(ii) Eigen values and Eigen vectors

(iii) Minimal polynomial.

(3) Prove that the orthogonal set of vectors in innerproduct space is linearly independents.

(4) If $T : R^3 \to R^2$

$T(e_1) = (1, -1),\ T(e_2) = (-2, 2), T(e_3) = (3, 5)$

$B_1 = \{(1, -1, 0),\ (0, 1, -1),\ (-1, 0, 1)\}$

$B_2 = \{(-1, 1), (-2, 5)\}$ are basis of R^3 and R^2 respectively. Then find $[T : B_1, B_2]$

(5) If T is invertible and λ is an eigen value of T, then show that λ^{-1} is eigen value of T^{-1}.
