HBY-1009 Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2015

Mathematics : CC-MATH - 401

(Advanced Calculus)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) The figures to the right indicate the marks of corresponding questions.

1 (a) Obtain the formula of radius of curvature of the curve $p = f(r)$. **6**

OR

(a) Obtain the formula of radius of curvature of the curve $y = f(x)$. **6**

(b) Attempt any **two** : **12**

(1) Find the radius of curvature of the curve $y = \frac{C}{2}\left(e^{x/c} + e^{-x/c}\right)$.

(2) If ρ_1 and ρ_2 are the radii of curvature at the extremities of the focal chord of parabola $\frac{a}{r} = 1 + \cos\theta$ then prove that $\frac{1}{\rho_1^{2/3}} + \frac{1}{\rho_2^{2/3}} = \frac{1}{a^{2/3}}$.

(3) Find the radius of curvature of curve $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$.

2 (a) Let $f: S \subset R^2 \to R$ be a continuous function on S which is bounded by $x = a,\ x = b,\ y = \phi(x)$, $y = \psi(x)$, where ϕ and ψ are continuous functions on $[a, b]$ such that $\psi(x) < \phi(x)$, $\forall\, x \in [a, b]$ then show that 6

$$\iint_S f\, dxdy = \int_a^b \int_{\psi(x)}^{\phi(x)} f\, dxdy$$

OR

(a) Prove that $\overline{|n}\,\overline{|n + \frac{1}{2}} = \frac{\sqrt{\pi}}{2^{2n-1}} \cdot \overline{|2n}$. 6

(b) Attempt any two : 12

(1) Change the order of integration of the double integration

$$\int_0^{2a} \int_{y^2/4a}^{3a-y} f\, dydx$$

(2) Evaluate : $\int_0^C \frac{x\, dx}{\sqrt{C^2 - x^2}}$

(3) By transforming into polar co-ordinates evaluate

$$\int_0^a \int_0^{\sqrt{a^2 - x^2}} \left(x^2 + y^2\right) dxdy$$

3 (a) State and prove Green's theorem. **6**

OR

(a) Prove that **6**

$$grad\left(\bar{f}\cdot\bar{g}\right)=\bar{f}\times curl\ \bar{g}+\bar{g}\times curl\ \bar{f}+\bar{f}\cdot(\nabla\bar{g})+\bar{g}\cdot\left(\nabla\bar{f}\right)$$

(b) Attempt any two : **12**

(1) If $\bar{f}=\left(x^3, y^3, z^3\right)$ and $\bar{\gamma}=(x, y, z)$ then prove that $cur\ \bar{f}=0$, $grad\left(div\ \bar{f}\right)=6\bar{r}$, $\nabla\left(div\ \bar{f}\right)=18$.

(2) Verify Green's theorem $\oint_C\left(y^2\,dx+x\,dy\right)$.

where C is the boundary of the square whose vertices are (0, 0), (2, 0), (2, 2), (0, 2).

(3) Verify divergence theorem

$$\iint_S\left[\left(x^3-yz\right)dydz-2x^2y\,dzdx+z\,dxdy\right]$$

where S is the surface of the cube with faces $x=0$, $x=a$, $y=0$, $y=a$, $z=0$, $z=a$.

4 Attempt any **two** : 8

(1) Obtain the formula of P-the length of perpendicular drawn from pole to a tangent to the curve $r = f(\theta)$.

(2) Prove that

$$\int_a^b (x-a)^{m-1} \cdot (b-x)^{n-1} \, dx = (b-a)^{m+n-1} \beta(m, n)$$

(3) Prove that

$$div\left(\overline{f} \times \overline{g}\right) = \overline{g} \cdot curl\ \overline{f} - \overline{f} \cdot curl\ \overline{g}$$

5 Attempt any **two** : 8

(1) Find the p-r equation of the curve $r = a(1 + \cos\theta)$.

(2) Evaluate : $\int_0^1 \int_x^{2x} x^2 y \, dxdy$

(3) Evaluate : $\int_0^\infty \frac{x^7\left(1-x^9\right)}{(1+x)^{25}} dx$

(4) Evaluate $\oint_C ((x+y)dx + (x-y)dy]$

where C is the boundary of region bounded by circle $x^2 + y^2 = 1$, line segments $y = 0$, $0 \le x \le 1$ and $x = 0$, $0 \le y \le 1$.