

P. S. SCIENCE & H. D. PATEL ARTS COLLEGE, KADI

Internal Examination

B. Sc. Semester - I

[Mark : 40

7-10-2015]

C. C. Mathematics - III

[1-30 to 3-00

1. [A] State and prove Leibnitz's theorem.

OR

[A] State and prove cauchy's mean value theorem.

[B] Any two

(1) If $y = (x + \sqrt{x^2 + 1})^m$ then prove that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

(2) Prove that $\sqrt{\frac{1-x}{1+x}} < \frac{\log(1+x)}{\sin^{-1}x} < 1, 0 \leq x \leq 1$

(3) Find co-efficient of x^4 in the expansion of $\log(\cos x)$

2. [A] Derive the Redicution for mula for $\int_0^{\pi/2} \cos^n x dx$

OR

[A] Derive the Reduction formula for $\int_0^{\pi/2} \sin^n x dx$

[B] Any two.

(1) Evaluate : $\int_0^1 x^4(2-x^2)^{3/2} dx$

(2) Find arc lenth of curve $y = \log(\cos x)$ between

$$x = 0 \text{ to } x = \frac{\pi}{4}$$

(3) Find surface area of cone whose base radius is r and height is h .

(1)

[P.T.O.]

3. [A] Prove $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

OR

- [A] Prove $\frac{l}{r} = 1 + \cos\theta$, where l is the length of semi-latus rectum.

[B] Any two.

- (1) Find orthogonal set of $\{(1, 1, -1), (1, -1, 1), (-1, 1, 1)\}$
 (2) Obtain polar eqⁿ of line passing through

$$A(-3, \frac{\pi}{2}), B(1, \frac{\pi}{3}) \text{ also find } p \text{ and } \alpha.$$

- (3) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r = |\bar{r}|$

4. [A] Obtain Necessary condition for the sphere S_1 and S_2 to be orthogonal.

OR

- [A] Obtain eqⁿ of Right circular cone where u is

$$\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n} \text{ and semi vertical Angle is } \nu.$$

[B] Any two.

- (2) If $2x - 3y + z = k$ is tangential plane to sphere $x^2 + y^2 + z^2 = 7$ then find k and co-ordinate of contact point.
 (2) Prove $xy + yz + zx = 0$ is Right Circular cone also find semi-vertical angle & eqⁿ of axis.
 (3) Find eqⁿ of cylinder whose axis is parallel to Z-axis and guiding curves are $x^2 + y^2 + z^2 = 8$, $x + 2y + 2z = 6$