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PP-457-458

Seat No.________

B. Sc. (Sem. VI) Examination
April / May - 2016
Mathematics : CC MATH - 603
603-A - General Topology
603-B - Number Theory

Time : 3 Hours] [Total Marks : **70**

603-A - General Topology

Instructions : (1) There are five questions.
(2) Figures to the right indicate marks of the corresponding question.

1 (a) Define closed set in a topological space : **6**
Prove that :
Finite union of a closed sets is a closed set and Arbitrary intersection of a closed sets is a closed set.

(b) Define closure of a subset of a topology space. **6**
If A be a subset of a topological space X and $x \notin \overline{A}$; then prove that $x \notin F$ for some closed set containing A.

(c) Let $T_1 = \{\phi, X, \{a\}, \{a,b\}\}$ and $T_2 = \{\phi, X, \{a\}, \{b, c\}\}$ **6**
are topologies on $X = \{a, b, c\}$.
Then (i) Find out the smallest topology containing T_1 and T_2.
(ii) Find out the largest topology contained in T_1 and T_2.

OR

1 (a) In a topological space (X, T); prove that $A \subset \overline{A}$, and $\overline{A \cup B} = \overline{A} \cup \overline{B}$, where $A \subset X$ and $B \subset X$. **6**

(b) In a topological space (X, T); prove that $O \subset X$ is open iff O is a neighborhood of each of its points. **6**

(c) Is the collection $T = \{G \subset \mathbb{R} / \mathbb{R} - G = \phi \text{ or } \mathbb{R} \text{ or infinite}\}$ a topology on $\mathbb{R}$? **6**

2 (a) Define interior of a subset of a topology space. If A be a subset of a topology space X; then prove that $C(Int.A) = \overline{C(A)}$ and $C(\overline{A}) = Int.(C(A))$. **6**

(b) Define continuity between two topological spaces. **6**

Prove that :

$f:(X,T) \to (Y,T')$ is continuous iff for each subset A of X; $f(\overline{A}) \subset \overline{f(A)}$.

(c) Let Y be a subspace of a topological space X and let A be a subset of Y. **6**

Then show that : $\overline{A}^{Y} \subset \overline{A}^{X}$

OR

2 (a) Let X and Y be topological spaces. **6**

Prove that :

$f: Y \to X$ is continuous iff

$f': Y \to f(Y)$ is continuous.

(b) If $f:(X,T)\to(Y,T')$ is continuous at $a\in X$; **6**

and if $g=(Y,T')\to(Z,T'')$ is continuous at $f(a)\in Y$; then show that $gof:(X,T)\to(Z,T'')$ is continuous at $a\in X$.

(c) Is a function $f:(-1,1)\to R$ homeomorphism ? Why ? **6**

3 (a) Let Y be a subset of a topological space X and let $T'=\{O'\subset y\}/O'=O\cap Y$; where O is an open set in X} **6**

Then prove that : T' is a topology on Y.

(b) Define connected subset of a topological space. **6**

Let $f:(X,T)\to(Y,T')$ is continuous function.

Prove that : If $A\subset X$ is connected, then $f(A)$ is connected.

(c) Let A and B be connected subsets of a topological space X. If $A\cap B\neq\phi$, then show that $A\cup B$ is connected. **6**

OR

3 (a) State and Prove : Intermediate Value Theorem. **6**

(b) State and Prove : Fixed Point Theorem. **6**

(c) Let $T=\{\phi, X, \{b\}, \{d,e\}, \{b,d,e\}, \{c,d,e,a\}\}$ **6**

be a topology on $X=\{a,b,c,d,e\}$.

Is X dis-connected space ?

Is $A=\{c,e,a\}$ connected subset of X ?

4 Attempt any **two** : 8

(a) Let $f:[a, b] \to R$ continuous.

If $f(a)\,f(b) < 0$; then show that there exist $x \in [a, b]$ s.t. $f(x) = 0$.

(b) Define inclusion map.

Let Y be a subspace of a topological space X. Then prove that :

The inclusion mapping $i: Y \to X$ is continuous.

(c) Prove that :

In a discrete topological space; each subset is simultaneously open and closed.

5 Attempt any **two** : 8

(a) Let $X = \{a, b, c, d, e\}$ be a topological space with topology $T = \{\phi, X, \{a, b\}, \{c, d, e\}\}$.

If $A = \{b, c, d\}$, then find out $Cl.(A)$; $Int.(A)$ and $Bdy.(A)$.

(b) If $T = \{\phi, X, \{2\}, \{4, 5\}, \{2,4,5\}, \{3,4,5,1\}\}$ be a topology on $X = \{1, 2, 3, 4, 5\}$; then find out all components of X.

(c) Let $f: X \to Y$ be any function. If (Y, I) is an indiscrete space; then show that :

$f:(X,T) \to (Y,I)$ is continuous for any T.

603-B - Number Theory

Instructions : (1) All questions are compulsory.

(2) Figure to the right indicates the marks of the corresponding question.

1 (a) State and prove the linear Diophantine equation. **7**

OR

(a) If a and b are positive integer then prove that $[a, b](a, b) = ab$. **7**

(b) Attempt any **three** : **12**

(1) Find the integers x and y satisfying $gcd(2947, 3997) = 2947x + 3997y$ by Euclidean Algorithm.

(2) Solve the Diophantine equation $158x - 57y = 7$.

(3) Show that fourth power of any integer is either of the form $5k$ or $5k+1$.

(4) Show that g.c.d. of $(a+b)$ and $\left(a^2 - ab + b^2\right)$ is 1 or 3.

2 (a) Prove that there are infinitely many prime of the from $4k+3$. 7

OR

(a) For arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n. 7

(b) Attempt any **three** : 12

(1) Find the last two digit of the number 7^{7^7}.

(2) State Chinese Remainder Theorem and Solve : $2x \equiv 1 \pmod{3}$, $3x \equiv 1 \pmod{5}$ and $5x \equiv 1 \pmod{7}$.

(3) Prove any prime of the form $3n+1$ is also of the form $6m+1$.

(4) Find the remainder when $2013^{1969} + 1969^{2013}$ number divided by 7.

3 (a) State and prove Wilson's Theorem. 7

OR

(a) Prove that the function ϕ is multiplicative. 7

(b) Attempt any three : 12

(1) Verify that $\phi(n) = \phi(n+1) = \phi(n+2)$, where $n = 5186$.

(2) Find last two digits of 3^{251} in its decimal represents.

(3) Prove that $2^{117} \neq 2 \bmod (117)$.

(4) By Euler's Theorem, for any integer a, $a^{37} \equiv a \pmod{1729}$.

4 Attempt any six : **13**

(1) Find the remainder when $1! + 2! + 3! + \ldots.. + 100!$ is divisible by 12.

(2) $\sqrt{p}$ is irrational for any prime p.

(3) Verify that $18^6 \equiv 1\left(\text{mod } 7^k\right)$, for $k = 1, 2$.

(4) Find last two digits of 7^{100} in its decimal represents.

(5) If $a|b^2$ then $a|b$? Justify whenever it must be true.

(6) Solve the linear congruence $25x \equiv 15(\text{mod } 29)$.

(7) Verify $\phi(m \cdot n) = \phi(m) \cdot \phi(n)$ holds when $m = 36$, $n = 10$.

(8) If n and $n + 2$ are pair of twin primes then $\phi(n+2) = \phi(n) + 2$.