PP-443

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Seat No.________

B. Sc. (Sem. VI) Examination

April / May – 2016

CC-MATH-601 : Mathematics

(Abstract Algebra)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1) This question paper contains FIVE questions and all questions are compulsory.

(2) Figures to the right indicate marks of each question.

1 (a) Define a ring. **6**

Suppose addition and multiplication are two binary operations defined on non-empty set R such that all the axioms for a ring except commutativity of addition are assumed. If there exists an element 1 such that $a \cdot 1 = 1 \cdot a = a$ for each $a \in R$, then show that commutativity of addition follows.

(b) Show that $(Z, +, \cdot)$ is a principal ideal ring. **6**

(c) Prove that the characteristics of an integral domain is either zero or prime number. **6**

OR

1 (a) A non-zero element [m] of a ring $(Z_n, +_n, \cdot_n)$ is a zero-divisor if and only if m and n are not relatively prime. **6**

(b) Give an example of a finite non-commutative ring without unity element. **6**

(c) If I_1 and I_2 are two ideals of a ring R then prove that $I_1 \cup I_2$ is an ideal of R if and only if either $I_1 \subset I_2$ or $I_2 \subset I_1$. **6**

2 (a) State and prove Eisenstein criteria. Deduce that $\sqrt[n]{P}, n \geq 2$ is always an irrational number for a prime P. **6**

(b) State and prove Gauss's Lemma. **6**

(c) If the degree of a polynomial $f(x) \in F[x]$ is n, then prove that $f(x)$ has at most n distinct roots in F. **6**

OR

2 (a) Define a degree of a polynomial. For non-zero polynomial f and $g \in D[x]$, in usual notations, prove that **6**

$[f{\cdot}g] = [f] + [g]$.

(b) State and prove the Division Algorithm for polynomials. **6**

(c) Find whether the polynomials **6**

(i) $x^2 + x + 1$ is irreducible over the field of integers mod 2 or not.

(ii) $x^3 - 9$ is irreducible over the field of integers mod 11 or not.

3 (a) Let R be a commutative ring with unity where **6**

$$R=\left\{\frac{p}{q} \mid p\in\mathbb{Z},\ q\in\mathbb{N}\ and\ q\ is\ odd\right\}.$$

Let I be a subset of R consisting of rational numbers with even numerators and odd denominators then prove that

(i) I is an ideal of R

(ii) R/I is a field.

(b) Prove that an ideal I=<P> is a maximal ideal of ring $(Z, +, \cdot)$ if and only if P is a prime integer. **6**

(c) In usual notations, if $\phi:(R, +, \cdot)\rightarrow(R', \oplus, \odot)$ is a homomorphism then prove that $\phi(I)$ is an ideal of $\phi(R')$ if I is an ideal of R. In this case, will $\phi(I)$ be an ideal of R' ? Why ? **6**

OR

3 (a) Suppose R is a ring with unity, For homomorphism $\phi:(R,+,\cdot)\rightarrow(R', \oplus, \cdot),\ \phi(1)\neq 0'$. If R' is an integral domain then prove that $\phi(1)$ is the unity element of R'. **6**

(b) If an ideal M in a commutative ring R with unity is a maximal ideal then prove that the quotient ring R/M is a field. **6**

(c) For ring $(Z_{12}, +_{12}, \cdot_{12})$, find its all prime and maximal ideals. **6**

4 Attempt any **two** : 8

(a) Define integral domain.

Is $\left(M_2(\mathbb{Z}), +, \cdot\right)$ an integral domain ? Justify your answer.

(b) Obtain an equation of degree 3 over Z_7 having three solutions 1, 2 and 6 in Z_7.

(c) What is the intersection of a left ideal and a right ideal in a ring R ? Justify your answer.

5 Attempt any **two** : 8

(a) Prove that a homomorphism defined on a field is either a zero homomorphism or one-one.

(b) Give an example of a division ring which is not a field.

(c) Find all zeros of polynomial $f(x) = x^2 - 5x + 6$ in $Z_{12}[x]$.

Why it violates the theorem "If degree of a polynomial $f(x) \in F[x]$ is n, then $f(x)$ has at most n distinct zeros in F." ?
