PO-409 Seat No.__________

B. Sc. (Sem. IV) Examination

April / May – 2016

Mathematics : CC - MATH - 402

(Advanced Linear Algebra)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are **compulsory**.

(2) Figures to the **right** side indicate the marks of questions.

1 (a) If $A, B \in M_{n\times n}$ are invertible then $(AB)^{-1} = B^{-1}A^{-1}$ and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$, where A^T is transpose of A. **8**

OR

(a) Explain : Row reduced echelon form and prove that an $n \times n$ matrix A is non singular iff its row reduced echelon form is I_n. **8**

(b) Attempt any **two** : **12**

(1) Determine whether the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$ is singular or non-singular.

(2) Solve the system of equations :

$2x+5y+2z-3t=3$

$3x+6y+5z+2t=2$

$4x+5y+14z+14t=11$

$5x+10y+8z+4t=4,$

by row reduction method.

(3) Obtain the linear transformation associated with the matrix $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$, where considering the standard bases of vector - spaces R^4 and R^3.

2 (a) State and prove Schwart'z inequality. 8

OR

(a) Let V be a finite dimensional vector space over the field R. If $x \in V$ be any vector then the function T_x on V^* defined by $T_x(f) = f(x), \forall f \in V^*$ is a linear functional on V^* also show that $x \to T_x$ is an isomorphism of V onto V^{**}. 8

(b) Attempt any **two** : 12

(1) If $\{f_1, f_2, f_3\}$ is dual basis of B where $B = \{(1, -1, 1), (1, 1, -1), (-1, 1, 1)\}$ is basis of R^3, then find $f_1(x), f_2(x)$ and $f_3(x)$; when $x = (1, 0, 0)$.

(2) Define : Inner product space. In an Inner product space V if $|\langle x, y\rangle| = \|x\| \|y\| \Leftrightarrow x$ and y are linearly dependent.

(3) If $B = \{x_1, x_2, \ldots\ldots, x_n\}$ is an orthonormal basis of n-dimensional inner product space V and $y \in V$ then $\sum_{i=1}^{n} |\langle y, x_i\rangle|^2 \leq \|y\|^2$.

3 (a) Let V be a vector space and $T \in L(V, V)$. The distinct eigen - vectors of T corresponding to distinct eigen values of T are linearly independent. 8

OR

(a) State and prove Cayley - Hamilton theorem. 8

(b) Attempt any **two** : 12

(1) Find eigen values and corresponding eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(2) Prove that : The characteristic equations of similar matrices are equal.

(3) Define : Eigen value of a linear transformation. If T is invertible and λ is an eigen value of T then show that λ^{-1} is an eigen value of T^{-1}.

4 Attempt any **five** : 10

(1) Explain : Rank and nullity of a matrix.

(2) If x and y are two vectors of inner product space V then show that $\|x\| = \|y\|$ iff $\langle x+y, x-y \rangle = 0$.

(3) Define : Characteristic polynomial and minimal polynomial.

(4) Suppose $T: R^2 \to R^2$ is defined by $T(\alpha, \beta) = (\alpha + 5\beta, 3\alpha + \beta) \in R$ then find T^*.

(5) Define : Invertible matrix and Transpose of a matrix.

(6) Let V be a vector space and $T \in L(V, V)$. If V_λ is the union of the set $\{0\}$ and the set of all eigen vectors of T corresponding to the eigen value λ on T then V_λ is a subspace of V.
