PO-407 Seat No.________

B. Sc. (Sem. IV) Examination

April / May - 2016

Mathematics : Paper - CC - MATH - 401

(Advanced Calculus)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are **compulsory**.
(2) The figures to the **right** indicate marks of the corresponding questions.

1 (a) Angle between radius vector and tangent to a continuous curve $r = f(\theta)$ is ϕ and θ is angle between radius vector and $\overrightarrow{OX}$ then prove that **5**

$$\tan\phi = r \cdot \frac{d\theta}{dr}.$$

OR

(a) Prove that **5**

$$\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}.$$

(b) Attempt any three : **15**

(1) Find the *p-r* equation of $r = a(1 - \cos\theta)$.

(2) Find the center of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$ and prove that evaluate of the curve is

$$(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}.$$

(3) Prove that

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi.$$

(4) Prove that $\int_0^{\infty} 3^{-4x^2}\, dx = \frac{\sqrt{\pi}}{4.\sqrt{\log 3}}$

2 (a) Prove that $\iint_S xy\, dx\, dy = \frac{a^4}{8}$. Where S is region of $x^2 + y^2 = a^2$ in first quadrant. 5

OR

(a) Prove that $\iint_S 8xy\, dx\, dy = \frac{32}{3}$ where S is region between $y^2 = 2x$ and $x^2 = 2y$. 5

(b) Attempt any **three** : 15

(1) Find the volume of a sphere $x^2 + y^2 + z^2 = a^2$ using double integral.

(2) Change the order of integration

$$\int_0^1 \left[\int_1^{x+1} f(x, y)\, dy \right] dx.$$

(3) Using polar co-ordinate, evaluate

$$\iint_R e^{-x^2-y^2}\, dx dy,\ R = \left\{(x, y) \middle| x^2 + y^2 \le a^2, x \ge 0, y, \ge 0\right\}$$

(4) Evaluate $\int_0^1 \int_1^{x^2} \int_{2y}^{x+y} x\, dx dy dz.$

2 (a) State and prove Stoke's theorem. **5**

OR

(a) State and prove Green's theorem. **5**

(b) Attempt any **three** : **15**

(1) Prove that

$$div\left(\phi \bar{f}\right) >= \not\supset \; div\,\bar{f} + \bar{f} \cdot grad\not\supset.$$

Where $\not\supset$ is a Scalar function and $\bar{f}$ is a vector function.

(2) Evaluate $\oint_C \left[\left(x^2+y^2\right)dx + \left(2x+y^2\right)dy\right]$.

Where $C:\square ABCD$ is square,

$A(1,1), B(1,2),\; C(2,2), D(2,1)$.

(3) Verify Green's theorem

$\oint_C \left[(2x-y)dx + (x+3y)dy\right]$ where C is the boundary of ellipse $x^2+4y^2=4$.

(4) If $\bar{F}=(ax, by, cz)$, a, b, c are constants then prove that

$$\iint_S \bar{F}\cdot\bar{n}\,ds = \frac{4\pi}{3}(a+b+c).$$

Where S is surface of sphere

$x^2+y^2+z^2=1$.

4 Attempt any **five** : **10**

(1) Find the radius of curvature of

$y = \log \sec x$ at (0, 0).

(2) Find $S = \psi$ equation of $y = C \cdot \cosh\left(\frac{x}{C}\right)$.

(3) Prove that

$\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$.

(4) Evaluate :

$$I = \int_0^5 \int_{\theta=0}^{2\pi} \int_{\not\supset=0}^{\pi} r^4 \sin \not\supset \; dr \cdot d\theta \cdot d\phi.$$

(5) Prove that :

$Curl\left(\bar{f} + \bar{g}\right) = Curl\,\bar{f} + Curl\,\bar{g}$.

(6) If $\bar{r} = (x, y, z)$ then prove that

$div\,\bar{r} = 3$ and $Curl\,\bar{r} = \bar{0}$.

(7) Find the area of ellipse

$\left.\begin{array}{l} x = a\cos\theta \\ y = b\sin\theta \end{array}\right\}$, using Green theorem.
