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PPD-1656 Seat No._____

B. Sc. (Sem. II) Examination

April / May - 2016

CC MAT-122 : Mathematics

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate the marks of the corresponding question.

1 (a) If n is any rational number then prove that **7**

$(\cos\theta + i\sin\theta)^n = \cos\theta + i\sin n\theta$, *where* $i = \sqrt{-1}$.

OR

(a) Expand $\sin n\theta$, $\cos n\theta$, $\tan n\theta$, in the power of $\sin\theta, \cos\theta, \tan\theta$ respectively, where $n \in N$.

(b) Attempt any two. **8**

(1) If $x = cisa$, $y = cis\beta$, $z = cisy$ and $x + y + z = o$ then prove that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.$$

(2) If $x^2 - 2\cos\theta + 1 = 0$ then prove that

$x^{2n} - 2x^n \cos n\theta + 1 = 0.$

(3) Find distinct possible values of $(-1)^{1/6}$

2 (a) State and prove that De'Almbert ratio test. 7

OR

(a) Find the real and imaginary part of

$(\alpha+i\beta)^{(x+iy)}$.

(b) Attempt any two : 8

(1) Discuss the convergence of

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots$$

(2) Prove that $\sinh^{-1} z = \log\infty\left(z+\sqrt{z^2+1}\right)$.

(3) Find modulus and argument of $(1+i)^{1-i}$

3 (a) Prove in usual notations : 7

$$\frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)}V.$$

OR

(a) Define linear differential equation and write the method of solving it.

(b) Solve any two Differential Equations : 8

(1) $\left(D^2-2D+2\right)y = e^x+\cos 2x.$

(2) $\frac{dy}{dx}+\frac{4xy}{x^2+1}=\frac{1}{\left(x^2+1\right)^3}.$

(3) $\frac{dy}{dx}=\frac{x^2-y^2}{2xy}.$

4 (a) If A and B are $m \times n$ symmetric matrices then prove that $AB - BA$ is a skew symmetric matrix. 7

OR

(a) If A and B are $m \times n$ and $n \times p$ matrices respectively then prove that $(AB)^T = B^T A^T$.

(b) Attempt any two : 8

(1) Find A^{-1} for the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

(2) For matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is Hermitian matrix or not?

(3) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then prove that $A^3 = A^{-1}$.

5 Attempt any five : **10**

(1) Find all positive distinct values of $(-1+i)^{2/5}$.

(2) Find real and imaginary part of e^{z^2} where $z = x + iy$

(3) Discuss the convergence of $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\ldots..$

(4) Find the radius of convergence of $\sum \frac{nx^n}{3^n}$.

(5) Find integrating factor of $y' - y \tan x = e^x$.

(6) Define transpose of matrix A. Find transpose of $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

(7) Define symmetric and skew symmetric matrix.
