HBX-956 Seat No.________

B. Sc. (Sem. II) Examination

April / May - 2015

Mathematics (Core Compulsory)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate marks of the question.

1 (a) State and prove De Moivre's theorem. **7**

OR

(a) If q is a positive integer then show that $(\cos\theta + i\sin\theta)^{1/q}$ has only q distinct values. **7**

(b) Answer the following : (any **two**) **8**

(1) If α and β are roots of equation $x^2 - 2x + 4 = 0$ then prove that

$$\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{3}.$$

(2) Find all possible values of $\left(-1+i\sqrt{3}\right)^{3/2}$.

(3) Prove that $\dfrac{1+\cos 7\theta}{1+\cos\theta} = \left(x^3 - x^2 - 2x + 1\right)^2$ where $x = 2\cos\theta$.

2 (a) Prove that 7

(i) $\cosh^{-1}(z) = \log\left(z + \sqrt{z^2 - 1}\right)$

(ii) $\tanh^{-1}(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$

OR

(a) Show that the series $\sum \frac{1}{n^p}$ is convergent for $p > 1$ and divergent for $p \le 1$. 7

(b) Answer the following : (any **two**) 8

(1) If $i^{\alpha + i\beta} = \alpha + i\beta$ then prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$, where n is any positive integer.

(2) Discuss the convergence of series

$$1 + \frac{1}{2}x + \frac{1}{5}x^2 + \frac{1}{10}x^3 + \dots\dots\dots + \frac{x^n}{n^2+1} + \dots\dots\dots$$

(3) Find the Radius of convergence and Interval of convergence of series

$$\sum_{n=1}^{\infty}\left(1 + \frac{1}{n}\right)^{\frac{n^2}{2}} \cdot x^n .$$

3 (a) Define Linear Differential Equation of first order and first degree and write the method of solving it. 7

OR

(a) Find the formula for $\frac{1}{D^2 + a^2}\cos ax$ and $\frac{1}{D^2 + a^2}\sin ax$. 7

(b) Answer the following : (any **two**) **8**

(1) Solve the differential equation

$$\left(xy+x^2y^3\right)\frac{dy}{dx}=1.$$

(2) Solve the differential equation

$$y\sqrt{1+p^2}=(x+yp)^2 \text{ where } p=\frac{dy}{dx}.$$

(3) Solve : $\left(D^2-6D+9\right)y=e^{3x}\left(20x^3+6x+1\right)$

4 (a) If A and B are matrices of the type $m\times n$ and $n\times p$ respectively then prove that **7**

$$(AB)^T=B^TA^T.$$

OR

(a) Prove that every square matrix can be represent unique as a sum of a symmetric and a skew symmetric matrix. **7**

(b) Answer the following : (any **two**) **8**

(1) Find A^{-1} for matrix $A=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 3\\ 1 & 2 & 4\end{bmatrix}$.

(2) Find the rank of matrix $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 5 & 7\\ 3 & 4 & 7 & 10\end{bmatrix}$

using Row-Reduction method.

(3) Solve the equations :

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$

by elementary row-operations.

5 Answer the following : (any **five**) **10**

(1) Simplify : $\dfrac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5}$

(2) Prove that $\tan 5\theta = \dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$ where $t = \tan\theta$

(3) Find the principal value of $\log(-i)$.

(4) Discuss the convergence of series $\sum_{n=1}^{\infty} \dfrac{\sqrt{n} - 1}{n^2 + 1}$

(5) Solve : $\dfrac{dy}{dx} + y\sec x = \tan x$.

(6) Solve : $2p + p^2 = \log(y - xp)$, where $p = \dfrac{dy}{dx}$.

(7) Define : Hermitian and Skew-Hermitian matrix.

(8) Find the rank of a matrix $\begin{bmatrix} 3 & 2 & 0 & -1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$.