GAF-475-476-477 Seat No.________

B. Sc. (Sem. V) Examination

November / December – 2015

Mathematics : CC-MATH : 504-(A), (B) & (C)

1. 504-(A) : Boolean Algebra
2. 504-(B) : Mechanics - I
3. 504-(C) : Operation Research - I

Time : 3 Hours] [Total Marks : 70

1. 504-(A) : Boolean Algebra

1 Define lattice.

(a) Let $\langle L,\leq\rangle$ be a lattice and a, b, c $\in$ L. Then prove that $a*(b\oplus c)\geq(a*b)\oplus(a*c)$.

OR

(a) State only modular inequality. Let $\langle L,\leq\rangle$ be a lattice and a, b $\in$ L. Then prove that $a\leq b\Leftrightarrow a\oplus b=b$.

(b) Attempt any **two** :

(i) Define equivalence relation with illustration.

(ii) Define lattice homomorphism. Prove that lattices $<P(\{a,b\}),\subseteq>$ and $\langle S_6,D\rangle$ are isomorphic.

(iii) Prove that every chain is a distributive lattice.

2 (a) Define complete lattice.

Let $\langle L,*,\oplus,0,1\rangle$ be a lattice and a, b, c, $\in$ L then prove that $a\oplus(b*c)=(a\oplus b)*(a\oplus c)$

OR

(a) Define bounded lattice. Give illustration of a lattice which is not complemented.

(b) Attempt any **two** :

(i) Prove that sublattice of a distributive lattice is also a distributive lattice.

(ii) State all the properties of Boolean Algebra.

(iii) Let $\langle B,*,\oplus,',0,1\rangle$ be a Boolean Algebra and a, b $\in$ L. Then prove that $a\oplus(a'*b)=a\oplus b$.

3 (a) Define a fundamental product with illustration.

OR

(a) Define minterm. Obtain product of sum canonical form of $\alpha(x_1,x_2,x_3)=(x_1\oplus x_2')'*x_3$.

(b) Attempt any **two** :

(i) Find the value of $\alpha(x_1,x_2,x_3,x_4)=x_1*x_2*[(x_1*x_4)\oplus x_2'\oplus(x_1'*x_3)]$ for $x_1=2$, $x_2=6$, $x_3=3$ *and* $x_4=6$ over the Boolean Algebra $\langle S_6,D\rangle$.

(ii) Prove that the Boolean expression

$\alpha\ (x, y, z) = (x * z) \oplus (x' * y)$ and

$\beta = (x, y, z) = (x \oplus y) * (x' \oplus z)$ are equivalent.

(iii) Minimize the following Boolean expression using Karnaugh map representation.

$\alpha(x_1, x_2, x_3, x_4, x_5) = \oplus(0,1,3,8,9,13,14,15,16,17,19,$

$24,25,27,31)$

4 Attempt any **four** :

(i) Draw Hasse diagram of a poset S_{24}.

(ii) Define lattice as an algebraic structure. Let $\langle L, *, \oplus \rangle$ be a lattice then prove that a*a=a for $\forall a, b \in L$

(iii) Define square free lattice with illustration.

(iv) Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean algebra and $a, b \in B$. then prove that $a \le b \Leftrightarrow a * b' = 0$

(v) Give illustration of not a symmetric Boolean expression.

2. 504-(B) : Mechanics - I

Instructions: (1) All questions are compulsory.
(2) Figures to the right indicate marks of the corresponding question.

1 (a) State and prove : Lami's theorem. 6

(b) Forces of magnitudes 3, 4, 5, kg. act at a point in directions parallel to the sides of an equilateral triangle taken in order. Find their resultant. 6

(c) Define : Axis of ; vertex of; span of; sag of ; parameter of; directrix of the catenary. 6

OR

(a) It two forces P and Q act at such an angle that their resultant is of magnitude P, show if P is doubled, Q remaining unaltered, then new resultant will be at right angles to Q and its magnitude will be $\sqrt{4P^2 - Q^2}$ 9

(b) A uniform beam of length 2a, rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the beam is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$. 9

2 (a) A weight K' is supported on a smooth plane of inclination α to the horizontal by a force whose line of action makes an angle 2α with the horizontal. If pressure on the plane be arithmetic mean of weight and the force. 9

Show that : $\sin(2\alpha) = \frac{3}{4}$.

(b) Define the mass of system of particles and prove that its unique exists. 9

OR

2 (a) A string ABCD is suspended from two points A and D. It carries weights 30 kg and K' kg respectively at B and C in it. The inclination to the vertical of AB is 30° and that of CD is 60°; the angle BCD is 120°. Find K' and tensions in different parts of the string. 9

3 (a) Determine the smallest angle for the equilibrium of a homogeneous ladder of a given length; the coefficient of friction for all surfaces being μ. 9

(b) For a common catenary; prove in usual notations $S = c\tan\psi \,\&\, S = c\sinh\left(x/c\right)$ 9

4 (a) A cable of length $2l$ hangs between two points A and B on the horizontal line. If the central dip of the cable is $\frac{l}{n}$; then show that the distance between the two supports is 9

$$AB = \ell\left(\frac{n^2-1}{n}\right)\frac{\log(n+1)}{\log(n-1)}$$

(b) Discuss the nature of equilibrium using Z-test.

3. 504-(C) : Operation Research - I

Instructions: (1) All questions are compulsory.
(2) Figure to the right indicate marks of the question.

1 (a) Use the graphical method to solve the following L.P. problem. 10

Minimize = $z = 20x_1 + 10x_2$

Subject to constraints

$x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

and $x_1, x_2 \geq 0$

(b) Solve the given L.P.P. by Simplex method : 10

Max $z = 4x_1 + 3x_2$

S.to c. $2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400, x_2 \leq 700$ and $x_1, x_2 \geq 0$

OR

(a) Prove that the sets of all feasible solutions of a linear programming problem is a convex set. 10

(b) Solve the given L.P.P., using Simplex method. 10

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

S.to c. $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2 \geq 0$

2 (a) Solve the following L.P.P. using two phase method. 10

Minimize $z = x_1 + x_2$

S.to c. $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

(b) Solve the following L.P.P. using Big M method. 10

Min $z = 3x_1 + 2x_2 + 3x_3$

S.to c. $x_1 + 4x_2 + x_3 \geq 7$

$2x_1 + x_2 + x_4 \geq 10$

and $x_1, x_2, x_3, x_4 \geq 0$

OR

2 (a) Solve the following L.P.P. using Big M method. 10

Maximize $z = 3x_1 - x_2$

S.to c. $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4,\ x_1, x_2 \geq 0$

(b) Solve the following L.P.P. using two phase method. 10

Maximize $z = -4x_1 - 3x_2 - 9x_3$

S.to c. $2x_1 + 4x_2 + 6x_3 \geq 15$

$6x_1 + x_2 + 6x_3 \geq 12$ and

$x_1, x_2, x_3 \geq 0$

3 (a) Find the dual of following L.P. problem : 10

Maximize $z = 3x_1 + 2x_2 - 4x_3 + x_4$

S.to c. $2x_1 + x_2 + x_3 - x_4 = 1$

$5x_1 - 2x_2 + 3x_3 + 4x_4 = 3$

and $x_1, x_2 \geq 0$, x_3 and x_4 are unrestricted.

(b) Solve the given L.P.P. using dual Simplex method. 10

Minimize $z = 2x_1 + 2x_2$

S.toc. $2x_1 + 4x_2 \geq 1$

$x_1 + 2x_2 \geq 1$

$2x_1 + x_2 \geq 1$, and $x_1, x_2 \geq 0$

OR

3 (a) Prove that the dual of a dual is primal. 10

(b) Solve the following Integer L.P. problem using Gemory's cutting plane method.

Maximize $z = x_1 + x_2$

S.toc. $3x_1 + 2x_2 \leq 5$

$x_2 \leq 2$

and $x_1, x_2 \geq 0$ and are integers.

4 (a) Attempt any **two** : 10

(1) Define : Degenerate solution, convex set.

(2) Use the graphical method to solve following L.P. problem.

Minimize $z = -x_1 + 2x_2$

S.toc. $-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

(3) Find the dual of following L.P. problem.

Maximize $z = x_1 + 2x_2 + x_3$

S.toc. $2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$, and $x_1, x_2, x_3 \geq 0$