GAF-467 Seat No._______

B. Sc. (Sem. V) Examination

November / December - 2015

CC-MATH-503 : Mathematics

(Differential Equations)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate the marks of the corresponding questions.

1 (a) Prove that : 6

$$\frac{1}{f(D)}\left[e^{ax}.V\right]=e^{ax}.\frac{1}{f(D+a)}\,V$$

Where a = Constant, V = Function of x.

(b) Solve : $\left(D^2-1\right)y=x^2.\cos x$ 6

(c) Solve : $\left(D^2-2D+1\right)y=xe^x.\sin x$ 6

OR

1 (a) If $f(D)=(D-a)^r\cdot D(D)$ then 6

prove that

$$\frac{1}{f(D)}e^{ax}=\frac{1}{D(a)}\cdot\frac{x^r}{r!}e^{ax},\ \phi(a)\neq 0$$

$$=\frac{x^r.e^{ax}}{f^r(a)}$$

(b) Solve : $\left(D^2-1\right) y=x^2 . \cos x$ **6**

(c) Solve : $\left(D^2+1\right) y=\sec^2 x$ **6**

2 (a) If the linear differentiate equation **6**

$$P_0 y^{(n)}+P_1 y^{(n-1)}+P_2 y^{(n-2)}+\ldots+P_n . y=P(x)$$

where $P_0, P_1, P_2, \ldots . P_n$ are junction of x, is exact differential equation then prove that

$$P_n+(-1) P_{n-1}^{(1)}+(-1)^2 . P_{n-2}^{(2)}+\ldots .+(-1)^n . P_0^{(n)}=0$$

(b) Obtain the first integral of **6**

$$x^2 y_3+x y_2+(2 x y-1) y_1+y^2+2 x=0$$

(c) Solve $y^{(2)}=\sec^2 y \tan y$ **6**

Given that $y=0, y^{(1)}=1$ when $x=0$

OR

2 (a) Solve : **6**

$$x y^{(3)}+\left(x^2+x+3\right) y^{(2)}+(4 x+2) y^{(1)}+2 y=0$$

(b) Find the first integral of **6**

$$y_1 y_2-y x^2 y_1=x y^2$$

(c) Solve : **6**

$$y(1-\log y) y^{(2)}+(1+\log y)\left\{y^{(1)}\right\}^2=0$$

3 (a) Solve : $xy^{(2)}-(2x-1)y^{(1)}+(x-1)y=0$ **6**

(b) Solve : $y^{(2)}-2\tan x.y^{(1)}-5y=0$ **6**

[By Normal Form]

(c) Solve : $x^2y^{(2)}-2x(1+x)y^{(1)}+2(1+x)y=x^3$ **6**

[By Variation of Parameter]

OR

3 (a) Solve : $\left(1-x^2\right)y^{(2)}+xy^{(1)}-y=x\left(1-x^2\right)^{3/2}$ **6**

(b) Solve : $\log x.y^{(2)}+\sin x.y^{(1)}-2\cos^3 x.y=2\cos^5 x$ **6**

[By Changing the indepent variable]

(c) Solve : $3x^2y^{(2)}+\left(2-6x^2\right)y^{(1)}-4y=0$ **6**

[By Factorisation of the operator]

4 Solve any four : **16**

(1) $\left(D^3-2D^2-4D+8\right)y=0$

(2) $\left(D^2-1\right)y=\left(1+e^{-x}\right)^{-2}$

(3) $x^2\ y^{(4)}+1=0$

(4) $y^{(2)} = \frac{1}{\sqrt{ay}}$

(5) $y^{(2)} - 4y^{(1)} + 4y = e^{x}.\sin x$

[By method of undetermined coefficients]

(6) $y^{(2)} + 4y = \sec^2 2x$

[By Variation of Parameter method]
