GAF-460

Seat No.________

B. Sc. (Sem. V) Examination

November / December - 2015

Mathematics : Paper : CC - Math - 502

(Mathematical Analysis-1)

Time : 3 Hours] [Total Marks : 70

Instructions :

(1) There are **five** questions.

(2) Figures to the right indicate marks of corresponding question.

1 (a) State and prove : Archimedean property of R. 6

(b) Define cut on Q : 6
Prove that : If α and β cuts on Q; then $\alpha + \beta$ is also a cut on Q.

(c) Show that : The equation $p^2 = 12$ is not satisfied by any rational p. 6

OR

1 (a) Prove that : Q is dense in R. 6

(b) Prove that : The order set R has the least-upper bound property. 6

(c) If Z is a complex number such that $|Z| = 1$; then find out the value of $|1+Z|^2 + |1-Z|^2$. 6

2 (a) Define countable set. 6

Prove that : If A be the set of all sequences whose elements are 0 and 1; then A is uncountable.

(b) Define limit point of a set in metric space. 6

Prove that : If p is a limit point of a set E; then every neighbourhood of p contains infinitely many points of E.

(c) Let N be the set of all positive integers. 6

Show that : N×N is a countable set.

OR

2 (a) Define compact subset of a metric space. 6

Prove that : Every K-cell in R^k is compact.

(b) Define connected subset of a metric space. 6

Prove that : $E \subset R^1$ is connected

iff $x \in E, y \in E,$ and $x < z < y$; then $z \in E.$

(c) Construct a bounded set of real numbers with exactly four limit points. 6

3 (a) Let $\{S_n\}, \{t_n\}$ are complex sequences; and 6

$$\lim_{n\to\infty} S_n = S; \quad \lim_{n\to\infty} t_n = t$$

Then prove that $\lim_{n\to\infty} S_n t_n = st$ and

$$\lim_{n\to\infty} \frac{1}{S_n} = \frac{1}{S};$$

where $S_n \neq 0, S \neq 0.$

(b) Prove that : $\lim_{n\to\infty} \sqrt[n]{p} = 1$; where p > 0. 6

(c) Prove that : 6

Convergence of $\{S_n\}$ implies convergence of $\{|S_n|\}$. Is the converse true ?

OR

3 (a) State and Prove : Ratio Test. 6

(b) Define absolute convergence of a series. 6

Prove that : If $\sum a_n$ converges absolutely; then $\sum a_n$ converges.

(c) Investigate the behavior (i.e. convergence or divergence) of $\sum\left(\sqrt[n]{n}-1\right)^n$. 6

4 Attempt any **two** : 8

(a) If $|x|<1$,, then $\lim_{n\to\infty} x^n = 0$.

(b) Let $E \subset R^K$, prove that : E is closed and bounded

iff

E is compact

(c) Let $r \in Q$ and let $r^* = \{p \in Q / p < r\}$.

Then prove that : r^* is a cut on Q.

and $r^* + s^* = (r+s)^*$; where $s \in \varphi$.

5 Attempt any two : 8

(a) Define complex number,

If C = The set of all complex numbers and

R = The set of all real numbers;

then show that $R \subset G$.

(b) Show that : R is uncountable set.

(c) Discuss the convergence of the following series :

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots$$
