GAE-409 Seat No.________

B. Sc. (Sem. III) Examination

November / December – 2015

Mathematics : CC - MATH - 301

(Calculus & Linear Algebra)

Time : 3 Hours] [Total Marks : 70

Instructions: (1) All questions are compulsory.

(2) Figures to the right side indicate the marks of corresponding questions.

1 (a) If function $z = f(x, y)$ defined on an open set $E \subset R^2$ is differentiable at point $(x, y) \in E$ then prove that its partial derivatives $f_x \& f_y$ exists at point (x, y). 6

Is converse of above statement true ? Justify your answer.

OR

(a) If $z = f(x, y)$ possess continuous partial derivatives in its domain and if $x = \phi(t)$ and $y = \psi(t)$ possess continuous derivatives in their domain $[a, b]$, then prove that 6

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

(b) Attempt any two. 12

(i) If $u = x^2 + y^2, v = 2xy$ and $z = f(u, v)$ then prove that,

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2(u^2 - v^2)^{1/2} \cdot \frac{\partial v}{\partial u}$$

(ii) If $u = f(r), r^2 = x^2 + y^2 + z^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$$

(iii) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}$$

2 (a) If $u=\phi(H)$ is a function of a homogenous function $H=f(x,y)$ of degree m whose partial derivatives of second order exist then prove that 6

(i) $$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=m\frac{F'(u)}{F(u)},\ F'(u)\neq 0$$
$$=G(u)\quad Say$$

(ii) $$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=G(u)[G'(u)-1]$$

where, $H=f(x,y)=\phi^{-1}(u)=F(u)$

OR

(a) Explain the Lagrange's method of undetermined multiplies to determine the extreme values of a function of n-variables. 6

(b) Attempt any two : 12

(i) Prove that the extreme value of $u = ax^2 + by^2 + cz^2$ subject to the conditions $x^2 + y^2 + z^2 = 1$, $lx + my + nz = 0$ is given by $\frac{l^2}{a^2 - u} + \frac{m^2}{b^2 - u} + \frac{n^2}{c^2 - u} = 0$

(ii) Expand $f(x, y) = x^2 y + y^3$ in power of $x - 2$ and $y - 1$.

(iii) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \quad \text{and}$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u\,(1 - 4\sin^2 u)$$

3 (a) Let A be a non empty subset of a vector space V. show that $[A]$ is the smallest subspace of V containing A. 6

OR

(a) Let $s = \{\bar{x}_1, \bar{x}_2, \ldots\ldots\ldots, \bar{x}_{m-1}, \bar{x}_m\}$ be a finite ordered set in a vector space V with, $\bar{x}_1 \neq \theta$. Also S is linearly independent then show that, vector $\bar{x}_i$, for some $i, 2 \leq i \leq m$ is a linear combination of preceding vectors $\overline{x_1}, \overline{x_2}, \ldots\ldots\ldots \bar{x}_{i-1}$. 6

(b) Attempt any two : 12

(i) $Let\, T : R^2 \to R^2, T(3,2) = (1,5), T(0,1) = (-1,2)$

then find $T(a,b)$, where $(a,b) \in R^2$.

(ii) If $T : R^4 \to R^3$ defined by

$T(a,b,c,d) = (a+b, b-d, c-d)$ then find $n(T)$ and $r(t)$.

(iii) Show that the set

S = {(1,0,0), (1,1,0), (1,1,1)} is a basis of R^3.

4 Attempt any four. 16

(i) If $z = f(ax+by) + g(ax-by)$ then prove that

$$b^2 \frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$$

(ii) If $x = r\cos\theta,\ y = r\sin\theta$ then prove that

$$\begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = r \text{ and } \begin{vmatrix} r_x & r_y \\ \theta_x & \theta_y \end{vmatrix} = \frac{1}{r}$$

(iii) Varify Euler's theorem for the function

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}, \quad (x,y) \neq (0,0)$$

(iv) Let $T : R^3 \to R^3$, $T(a, b, c) = (2b,\ b+c,\ 3b-c)$ then show that T is a linear transformation.

(v) If $f(x,y)=\frac{xy}{x^2+y^2}, \quad (x,y)\neq(0,0)$

$$=0, \qquad (x,y)=(0,0)$$

then show that f is discontinuous at point (0,0).

(vi) If $u = x\log y + y\log x$ then prove that

$u_{xy} = u_{yx}$.