* K A I - 1 2 6 3 *

KAI-1263

Seat No.________

B. Sc. (Sem. IV) Examination

April / May – 2013

Mathematics

(CC - Maths - 402)

(New Course)

Time : 3 Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.
(ii) Figure in the right side indicate the marks of questions.
(iii) This question paper contain five questions.

1 (a) If A and B are two matrices of type $m \times p$ and $n \times p$ respectively. Then prove that $(AB)^T = B^T A^T$. **7**

OR

(a) Prove that an $n \times n$ matrix A is invertible if and only if the corresponding linearmap T of A with respect to standard basis is non singular.

(b) Attempt any two : **8**

(1) Let $T : R^3 \to R^2$ be linear transformations defined by $T(x, y, z) = (x + y, y + z)$ $B_1 = \{(1,1,0), (2,1,0), (0,0,1)\}$ and $B_2 = \{(1,1), (2,3)\}$ are ordered basis of R^3 and R^2 then find $[T : B_1, B_2]$

(2) Find matrix representation of linear map $T: R^3 \to R^2$ given by $T(x,y,z) = (z, y+z, x+y+z)$ relative to basis $\{(1,0,1), (-1,2,1), (2,1,1)\}$ of R^3.

(3) Determine the rank of matrix

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -2 \end{bmatrix}$$ by row reduction echelon form.

2 (a) Define bilinear form. Which of the following f defined R^2 are bilinear forms ? 7

(1) $f(\bar{x}, \bar{y}) = x_1 y_2 - x_2 y_1$

(2) $f(\bar{x}, \bar{y}) = (x_1 - y_1) + x_2 y_2$

Where $\forall \bar{x} = (x_1, x_2),\ \bar{y} = (y_1, y_2) \in R^2$

OR

(a) Let V be n–dimensional vector space and $B = \{x_1, x_2, \ldots\ldots, x_n\}$ be basis of V. Then there is uniquely determined basis $B^* = \{f_1, f_2, \ldots\ldots, f_n\}$ of V^* such that $f_i(x_j) = \delta_{ij}$ for $i, j = 1, 2, \ldots\ldots, n$. 7

(b) Attempt any two : 8

(1) Find dual basis for the basis $\{(1,-2,3), (1,-1,1), (2,-4,7)\}$ of R^3.

(2) $B = \{(1,1,1), (1,1,-1), (1,-1,-1)\}$ is basis of R^3. If $\{f_1, f_2, f_3\}$ is dual basis of B and if $x = (0,1,0)$. Then find $f_1(x), f_2(x), f_3(x)$.

(3) For linear map

$T: R^2 \to R^3$, $T(x, y) = (x+2y, x-y)$ find T^*.

If $\alpha = (1,3)$ find $T^*(\alpha)$.

3 (a) If X and Y are two vectors of inner product space $V(R)$ then show that $\|x\| = \|y\| \Leftrightarrow x+y$ and $x-y$ are perpendicular. 7

OR

(a) State and prove Schwartz's inequility. 7

(b) Attempt any two : 8

(1) By using Gram Schmit process obtain the orthonormal basis from the basis $\{(1,0,1),(1,0,-1),(0,3,4)\}$.

(2) Find orthonormal basis of $V_3(R)$ with standard inner product by using Gram Schmidt orthogonalization to vectors $\alpha_1 = (1,0,1)$, $\alpha_2 = (1,2,-2)$, $\alpha_3 = (2,-1,1)$.

(3) If x and y are vector in real inner product space then prove that $|\langle x, y\rangle| = \|x\|\|y\| \Leftrightarrow$ x and y are linearly dependent vectors.

4 (a) Prove that 7

The eigen vector of a symmetric linear map corresponding to different eigen values are perpendicular to each other.

OR

4 (a) State and prove Cayley Hamilton Theorem.

(b) Attempt any **two** : 8

(1) Find the eigen values and corresponding eigen vectors of matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

(2) By using Cayley Hamilton theorem find the inverse of matrix $A=\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 2\\ 0 & 0 & 3\end{bmatrix}$.

(3) If T is invertible and 'λ' is an eigen value of T then show that λ^{-1} is eigen value of T^{-1}.

5 Attempt any **five** : **10**

(1) Explain :
Dual basis, Inner product space.

(2) In a vector space R^2
for $X=(x_1, x_2), Y=(y_1, y_2)$
define $(X, Y)=x_1y_1+x_1y_2+x_2y_1-3x_2y_2$.
Then show that $\langle x, y\rangle$ is inner product in R^2.

(3) Prove that $\det\begin{bmatrix}x & y & y & y\\ x & y & x & x\\ x & x & x & y\\ y & y & x & y\end{bmatrix}=(x-y)^4$.

(4) $T: R^2 \to R^2$ is defined by
$T(\alpha, \beta)=(\alpha+5\beta, 3\alpha+\beta)\in R$ then find T^*.

(5) Find characteristic equation of matrix
$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$.

(6) Prove that
The orthogonal set of vectors in inner product space is linearly independent.