* K A I - 1 2 5 9 *

KAI-1259

Seat No.________

B. Sc. (Sem. - IV) Examination

April/May - 2013

Mathematics : Paper - CC-MATH-401

(Calculus)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figure in the right side indicate the marks of question.
(3) This question paper contains five questions.

1 (a) Find the formula of radius of curveture of the curve $x = f(t),\ y = g(t)$. **7**

OR

(a) Find the formula of centre of curvature of the curve $y = f(x)$ at $p(x, y)$.

(b) Attempt any **two**. **8**

(1) Find the radius of curveture of catenuras $y = \cos b(x)$.

(2) Find the p-r equation of $r^n = a^n \sin\theta$.

(3) Find the radius of curveture of the curve $r = a(1 - \cos\theta)$.

2 (a) Prove that $\beta(m,n) = \frac{\sqrt{m}\cdot\sqrt{n}}{\sqrt{m+n}}$. **7**

OR

(a) Prove that $\int_0^1 \frac{x^n}{\sqrt{1-x^2}} dx = \frac{1\cdot 3\cdot 5\ldots(n-1)}{2\cdot 4\cdot 6\ldots n}\cdot\frac{\pi}{2}$

where n=*even integer.*

(b) Attempt any **two**. 8

(1) Evaluate : $\int_0^1 x^6 \cdot \left(1-x^2\right)^{\frac{1}{2}} dx$.

(2) Prove that $\int_0^\infty x^2 \cdot e^{-b^2 x^2} dx = \frac{\sqrt{\pi}}{4b^3}$.

(3) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$.

3 (a) Find the volume of sphere $x^2+y^2+z^2=a^2$ using double integeral. 7

OR

(a) Find the volume bounded by cylindar $x^2+y^2=a^2$ and $x^2+z^2=a^2$ between plans $x=0$ and $x=a$.

(b) Attempt any **two**. 8

(1) Prove that $\iint_s xy\,dx\,dy = \frac{a^4}{8}$ where s is region of $x^2+y^2=a^2$ in first quavdrant.

(2) Charge the order of integration $\int_0^1 \left(\int_1^{x+1} f(x,y)\, dy \right) dx$.

(3) Using Polar co-ordinet, evaluate

$$\iint_R e^{-x^2-y^2}\,dxdy \text{ where}$$

$$R=\left\{(x,y)\Big/ \begin{matrix} x\geq 0, y\geq 0 \\ x^2+y^2\leq a^2 \end{matrix}\right\}.$$

4 (a) State and prove Green's theorem. **7**

OR

4 (a) State and prove Gauss theorem for divergence.

(b) Attempt any **two**. **8**

(1) If $\overline{F}=(ax,by,cz)$, a, b, c are constant then prove that $\iint_s \overline{F}\cdot\overline{n}ds=\frac{4}{3}\pi(a+b+c)$, where s is surface of sphere whose radius is *1*.

(2) Using Stone's theorem prove that $curl(grad\phi)=\theta$.

(3) If $\overline{r}=(x,y,z)$, $|\overline{r}|=r$ and $\phi(r)$ is a differential function of r then prove that $grad\phi(r)=\frac{\phi'(r)}{r}\overline{r}$.

5 Attempt any **five**. **10**

(1) Find the radius of curveture of curve $y=\log\sec x$ to *0(0,0)*.

(2) Find the formula of Polar subtangent and subnormal.

(3) Evaluate : $\int_0^\infty x^6\cdot e^{-2x}dx$.

(4) Using the formula $\overline{|n+1} = n\overline{|n}$ prove that $\overline{\left|\frac{-5}{2}\right.} = \frac{-8\sqrt{\pi}}{15}$.

(5) Evaluate : $\int_0^{\pi/2}\int_{-1}^{1}\left(x\sin y - ye^x\right)dydx$.

(6) If $\overline{f} = \left(x^2y, y^2z, z^2x\right)$ then find $div\,\overline{f}$.

(7) If $\overline{r} = (x, y, z)$ then prove that $curl\,\overline{r} = \theta$,