GAD-2325 Seat No.________

B. Sc. (Sem. V) Examination

November / December – 2013

Mathematics : Paper - CC - MATH - 504 B

(Mechanics - I)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate marks of the corresponding questions.

1 (a) State and prove Lami's theorem. **6**

(b) The magnitude of two forces P, Q acting at an angle θ is equal to $(2m+1)\sqrt{P^2+Q^2}$, where they act at an angle $\frac{\pi}{2}-\theta$ it is equal to $(2m-1)\sqrt{P^2+Q^2}$, show that $\tan\theta=\frac{m-1}{m+1}$. **6**

(c) If two forces P and Q act at such angle that their resultant is of magnitude P, show if the force P is doubled, Q remaining unaltered, then new resultant will be at right angles to Q and its magnitude will be $\sqrt{4P^2-Q^2}$ **6**

OR

1 (a) Define gradient of a scalar function. Prove that the component of a grad V in any direction is the rate of change of V in the direction. **6**

(b) If two forces of magnitudes P and Q are acting at angle θ to each other and the angle between the force $\overline{P}$ and resultant $\overline{R}$ is α then prove that 6

(i) $R^2=P^2+Q^2+2PQ\cos\theta$

(ii) $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$ where $P\left|\overline{P}\right|, Q=\left|\overline{Q}\right|, R=\left|\overline{R}\right|$

(c) Forces of magnitudes 3,4 and 5 kg wt. act at a point in directions parallel to the sides of an equilateral triangle taken in order. Find their resultant. 6

2 (a) Prove that in the usual notation that for an infinitesimal displacement of a rigid body in a plane, $\delta W = x\,\delta a + y\,sb + NS\theta$ and deduce the conditions of equilibrium. 6

(b) A weight W is supported on a smooth plane of inclination α to the horizontal by a force whose line of action makes an angle 2α with the horizontal. If pressure on the plane be arithmetic mean of weight and the force, show that $\alpha = \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$. 6

(c) Point O is the in centre of a triangle ABC. Then forces P, Q, R act on the direction $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. If the system is in equilibrium then prove that 6

$$\frac{P}{\cos A/2} = \frac{Q}{\cos B/2} = \frac{R}{\cos C/2}$$

OR

2 (a) State the principle of virtual work and using it to obtain the sufficient conditions for the equilibrium of a rigid body movable paralled to a fixed plane. 6

(b) A light rigid rod of length 2b terminated by heavy particles W_1 and W_2 is placed inside a smooth hemispherical bowl of radius a. Which is fixed with its rim horizontal. If the particle of weight W_1 rests just below the rim of the bowl, prove that **6**

$W_1 a^2 = W_2 (2b^2 - a^2)$

(c) A string ABCD is suspended from two points A and D. It carries weights of 30 kg and W kg respectively at B and C in it. The inclination to the vertical of AB is 30° and that of CD is 60°, the angle BCD is 120°. Find W and tensions in different parts of the string. **6**

3 (a) Define the mass of a system of particles and prove that it unique exist. **6**

(b) A sphere of radius 3 cm is cut off from a larger sphere of radius 10 c.m. the distance between their centres is 5 c.m. Locate the mass centre of the remaining volume. **6**

(c) A cable 200 ft. long hungs between two points at the same height. The sag is 20 ft and the tension at either point of suspension is 130 unit. Find total weight of cable. **6**

OR

3 (a) For a common catenary, prove that in usual notation. **6**

(i) $S = c \tan \psi$

(ii) $S = c \sin h\left(\frac{x}{c}\right)$

(b) Determine the smallest angle for the equilibrium of a homogeneous ladder of a given length, the coefficient of friction for all surfaces being μ. **6**

(c) Prove that the mass centre of a solid homogeneous hemisphere is at a distance $\frac{3a}{8}$ from the centre of its base, where a is the radius of the sphere. 6

4 Attempt any **two** : 16

(i) A weight is supported on a smooth inclined plane of inclination α, by a string inclined to the horizontal at an angle β. If the slope of the plane be increased to γ, the direction of the string remaining unaltered, the tension of the string is doubled, prove that $\cot\alpha = \tan\beta + 2\cot\gamma$.

(ii) Two heavy particles of weights W and W' are connected by a light inextensible string and hang over a fixed smooth circular cylinder of radius a, the axis of which is horizontal, when the system is in equilibrium, prove that $\frac{\sin\theta}{\sin\theta'} = \frac{W'}{W}$; where θ, θ' are the inclinations to the vertical of radii drawn to the particles W and W' respectively.

(iii) A cable of length $2l$ hangs between two points A and B on the horizontal line. If the central dip of the cable is $\frac{l}{n}$, then show that the distance between the two supports is

$$AB = l\left(n - \frac{1}{n}\right)\log\left(\frac{n+1}{n-1}\right)$$