GAD-2317 Seat No.________

B. Sc. (Sem. V) Examination

November / December – 2013

Mathematics : CC-MAT-503

(Differential Equations)

Time : 3 Hours] [Total Marks : 70

Instructions : (1) All question are compulsory.
(2) Figures to the right indicate marks of the question.

1 (a) If $f(D)=(D-a)^r \cdot \phi(D)$ then prove that 6

$$\frac{1}{f(D)}e^{ax}=\frac{1}{\phi(a)}\cdot\frac{x^r}{r!}e^{ax}, \phi(a)\neq 0$$

$$=\frac{x^r \cdot e^{ax}}{f^{(r)}(a)}$$

(b) Solve : $\left(D^2-3D+2\right)y=\sin\left(e^{-x}\right)$ 6

(c) Solve : $\left(D^2-1\right)y=\left(1+e^{-x}\right)^{-2}$ 6

OR

1 (a) Prove that $\frac{1}{f(D)}\left(e^{ax}, V\right)=e^{ax}\cdot\frac{1}{f(D+a)}\cdot V$ 6

where $a=$ constant, $V=$ function of x.

(b) Solve : $\left(D^3+1\right)y=\left(e^x+1\right)^2$ 6

(c) Solve : $\left(D^2-4D+4\right)y=x^2+e^x+\cos 2x$ 6

2 (a) Show that the equation 6

$$x^2y^{(3)}+4xy^{(2)}+\left(x^2+2\right)y^{(1)}+3xy=2$$

becomes exact on being multiplied by some power of x and obtain its first integer.

(b) Solve : $y^{(2)}=x^2\cdot\sin x$ 6

(c) Solve : $y^{(2)}+\cos ec^2 y\cdot\cot y=0$, give that 6

$y=\dfrac{\pi}{2}, y^{(1)}=1$ when $x=0$.

OR

2 (a) Solve : $y^{(2)}+2\tan x\cdot y^{(1)}+3y=\tan^2 x\cdot\sec x$ 6

(b) Solve : $\left(1+x^2\right)y^{(2)}+\left[1+\left(y^{(1)}\right)^2\right]=0$ 6

(c) Solve : $xy^{(2)}-y^{(1)}=x^3\cdot e^{x^2/2}$ 6

3 (a) Solve : $xy^{(2)}-(2x-1)y^{(1)}+(x-1)y=0$ 6

(b) Solve : $y^{(2)}-2\tan x\cdot y^{(1)}+5y=e^x\cdot\sec x$ 6

[by Normal from]

(c) Solve : $\cos x\cdot y^{(2)}+\sin x\cdot y^{(1)}-2\cos^3 x\cdot y=2\cos^5 x$ 6

[by changing the independent variable]

OR

3 (a) Solve : $\left(1-x^2\right)y^{(2)}+xy^{(1)}-y=\left(1-x^2\right)^{3/2}$ 6

(b) Solve : $y^{(2)}-4xy^{(1)}+\left(4x^2-1\right)y=-3\cdot e^{x^2}\sin x$ 6

[by using Normal from]

(c) Solve : $y^{(2)}-2y^{(1)}+y=x\cdot e^{x}\cdot Log\, x$ 6

[by variation of parameter method]

4 Solve any **four** : 16

(1) $\left(D^4-4D^3+8D^2-8D+4\right)y=0,\ D=\dfrac{d}{dx}$

(2) $\left(D^6-a^6\right)y=0,\ D=\dfrac{d}{dx}$

(3) $\sqrt{\left(a^2-x^2\right)^3}\cdot y^{(2)}=x$

(4) $ay^{(3)}=y^{(2)}$

(5) $y^{(3)}+x\cdot y^{(3)}-y^{(2)}=0$

(6) $y^{(2)}+2y^{(1)}+2y=1+x^2$

[by method of undetermine coefficients]