GAD-2310 Seat No.________

B. Sc. (Mathematics) (Sem. V) Examination

November / December – 2013

CCMAT-502 : Mathematical Analysis - I

Time : 3 Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory, there are five question.

(2) Figures to the right indicate marks of the corresponding question.

1 (a) State and prove : Archimedean Property of R. **6**

(b) State and prove : Schwarz Inequality for complex numbers. **6**

(c) Let $\bar{a} \in R^3, \bar{b} \in R^3$. Find $\bar{c} \in R^3$ and $r > 0$ such that $|\bar{x} - \bar{a}| = 2|\bar{x} - \bar{b}|$ if and only if $|\bar{x} - \bar{c}| = r$. **6**

OR

1 (a) Define ordered set. **6**

Prove that : The ordered set R has the least-upper-bound property.

(b) Prove that : The set of all rational number Q is dense in R. **6**

(c) Show that : $\beta = \{p \in Q / -p - r \notin 2^*;\ \text{for some } r > 0\}$ is a cut on Q. **6**

2 (a) If $\{E_n\}_{n=1}^{\infty}$ be a sequence of countable sets and if $S = \bigcup_{n=1}^{\infty} E_n$; then prove that S is countable. 6

(b) Prove that : Every K-Cell in R^k is compact. 6

(c) Define algebraic number. Show that : The set of all algebraic numbers is countable. 6

OR

2 (a) Define connected subset in a metric space. Prove that : 6

$E \subset R^1$ is connected iff

if $x \in E$, $y \in E$ and $x < z < y$; then $z \in E$.

(b) Prove that : Compact subsets of metric spaces are closed. 6

(c) Show by an example that the union of an infinite collection of closed sets need not be closed. 6

3 (a) Define Cauchy sequence in a metric space. Prove that : 6

In R^k, every Cauchy sequence converges.

(b) If $p > 0$ and $\alpha \in R$; then show that 6

$$\lim_{x \to \infty} \frac{n^{\alpha}}{(1+p)^n} = 0$$

(c) If $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + \sqrt{a_n}}$; where $n = 1, 2, 3, \ldots\ldots$ then prove that sequence $\{a_n\}_{n=1}^{\infty}$ converges. 6

OR

3 (a) State and prove : ROOT TEST. 6

(b) Prove that : $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. 6

(c) Discuss the convergence of the following series. 6

$$\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{3^n} \right)$$

4 Attempt any **two** : 8

(1) Show that : $p^2 = 2$ is not satisfied by any rational p.

(2) If z and w are complex; then show that $|z + w| \leq |z| + |w|$ and $|zw| \leq |z|\,|w|$.

(3) If N be the set of all positive integers and Z be the set of all integers; then show that N and Z have the same cardinal number.

(4) Find the radius of convergence of power series :

$$2z + 2z^2 + \frac{4}{3}z^3 + \frac{2}{3}z^4 + \frac{4}{15}z^5 + \ldots\ldots\ldots\ldots$$

5 Attempt any **two** : 8

(1) If $r, s \in Q$; then prove that $(r + s)^* = r^* + s^*$.

(2) Prove that : Every infinite subset of a countable set A is countable.

(3) If $\{S_n\}$ be a sequence of complex numbers and $\lim_{n\to\infty} S_n = S$; then prove that

$$\lim_{n\to\infty} (S_n)^{-1} = (S)^{-1}; \text{ where } S_n \neq 0 (n = 1, 2, 3, ...)$$

and $S \neq 0$.

(4) Prove that $\sum_{n=0}^{\infty} x^n$ converges if $0 \leq x < 1$ and diverges if $x \geq 1$.
