GAS-3159 Seat No.________

B. Sc. (Sem. III) Examination

November/December – 2013

Calculus & Linear Algebra : Paper - CC-MAT-301

(New Course)

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figure in the right side indicate the marks of the question.

1 (a) If $z = f(x, y)$ possess continuous partial derivatives in its domain and if the functions $x = \phi(t)$, $y = \psi(t)$ possess continuous derivatives in their domain $[a, b]$ then prove that : 8

$$\frac{dz}{dt} = \frac{dz}{dx}.\frac{dx}{dt} + \frac{dz}{dy}.\frac{dy}{dt}.$$

OR

(a) State and prove Young's theorem. 8

(b) Attempt any **two** : 12

(1) Define limit of a function of two variables using definition show that

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2} \text{ does not exist.}$$

(2) If $z = f(x, y)$ and $u = lx + my$, $v = ly - mx$ then prove that :

$$\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = (l^2 + m^2)\left(\frac{d^2z}{du^2} + \frac{d^2z}{dv^2}\right)$$

where l, m are constants.

(3) If $F(x, y, u, v) \equiv x^3 + y^3 + u^3 + 2v^3 - 5 = 0$

and $G(x, y, u, v) \equiv 2x^3 - y^3 + 3u^3 - v^3 - 7 = 0$

then find $\frac{d^2u}{dx^2}$ and $\frac{d^2x}{du^2}$.

2 (a) State and prove Euler's theorem. 8

OR

(a) If $u = \phi(H)$ is a function of a homogeneous function $H = f(x, y)$ of degree m whose partial derivatives of second order exists then prove that :

(i) $$x\frac{du}{dx} + y\frac{du}{dy} = m\frac{F(u)}{F'(u)},\ F'u \neq 0.$$

(ii) $$x^2\frac{d^2u}{dx^2} + 2xy\frac{d^2u}{dudy} + y^2\frac{d^2u}{dy^2} = G(u)(G'(u) - 1)$$

where $G(u) = m\frac{F(u)}{F'(u)}$

$H = f(x, y) = F(u) = \phi^{-1}(u)$.

(b) Attempt any **two** : **12**

(1) Show that of all triangles having given perimeter the largest in an equilateral triangle.

(2) State Taylor's theorem.

Expand $f(x,y)=x^2y+3y-2$ in power of $x-1$ and $y+2$.

(3) If $u=f(v)$, where v is homogenous function of x and y of degree n then prove that $x\frac{du}{dx}+y\frac{du}{dy}=nvf'(v)$.

3 (a) Let A be a non-empty subset of a vector space V. show that [A] is the smallest subspace of V containing A. **8**

OR

(a) If $B=\{\overline{u_1},\overline{u_2},......\overline{u_n}\}$ is a basis for vector space U and $\overline{v_1},\overline{v_2},......,\overline{v_n}$ be n-vectors in vector space V then show that there exists a unique linear transformation $T:U\to V$ such that $T(\overline{u_i})=\overline{v_i}$, for i = 1, 2, 3....... n. **8**

(b) Attempt any **two** : **12**

(1) Find $R(T)$, $N(T)$, $r(T)$ and $n(T)$ for the linear transformation $T:R^2\to R^3$, where $T(x,y)=(x,x+y,y)$.

(2) Let $U=\{a_1,a_2,a_3,a_4)|\,a_1+a_2=1=a_3+a_4\}\subset R^4$ then find $dim U$.

(3) $TR^2\to R^3$ be such that $T(1,3)=(3,-1,5)$ and $T(0,1)=(2,1,-1)$ then find $T(a,b)$.

4 Attempt any **five** : **10**

(1) If $u = x + y + z$, $v = x^2 + y^2 + z^2$,

$w = x^3 + y^3 + z^3 - 3xyz$ then prove that

$\frac{d(u,v,w)}{d(x,y,z)} = 0$.

(2) Varify Euler's theorem for the function

$f(x,y) = x^2 \tan^{-1} y/x - y^2 \tan^{-1} x/y$.

(3) Find the equation of the tangent plane and the normal to the surface

$4x^2 - 5y^2 + 7z^2 + 13 = 0$ at point $P(-1,4,-3)$.

(4) $T : R^3 \to R$ defined as $T(x,y,z) = 2x^2 - y - z$ show that T is not linear transformation.

(5) Show that the set $\{(1,0,0), (1,1,0), (1,1,1)\}$ is a basis of R^3.

(6) If $z = f(ax+by) + g(ax-by)$ then prove that

$b^2 \frac{d^2z}{dx^2} = a^2 \frac{d^2z}{dy^2}$.

(7) If $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$, $x + y \neq 0$ then prove that

$x\frac{du}{dx} + y\frac{du}{dy} = \tan u$.
