AAC-2125

AAC-2125 Seat No.________

M. Sc. (Sem. IV) Examination

April - 2019

Mathematics : MCB - 5

(Integral Trasforms)

Time : **2** Hours] [Total Marks : **35**

Instructions : (1) All questions are compulsory.
(2) Standard notations and conventions are followed.

1 Answer the following : (any ~~three~~ two) **7**

(1) Define a periodic function and prove that if

$$f(t+T) = f(t) \text{ then } L[f(t) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

(2) Find $L[f(t)]$ *if* $f(t) = \begin{Bmatrix} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{Bmatrix}$ and $f(t+2\pi) = f(t)$.

(3) If $L\{f(t)\} = \overline{f}(s)$ then prove that

$$L\left\{t^n f(t)\right\} = -1^n \frac{d^n}{dx^n}\left\{\overline{f}(s)\right\}$$

2 Answer the following : (any ~~three~~ two) **7**

(1) State and prove convolution theorem for Laplace Transforms..

(2) Find $L^{-1}\left\{\dfrac{2x^2+5s+7}{(s-2)(s^2+4x+13)}\right\}$.

(3) Apply Laplace Tansform to solve $y''' + 2y'' y' - 2y = 0$ subject to condition $y(0) = 0, y'(0) = 0$ and $y''(0) = 6$.

3 Answer the following : (any **two**) 7

(1) Define Fourier Transform of f(x) and find Fourier Transform of $f(x) = e^{-ax^2}$.

(2) Using Fourier transform solve ordinary differential equation $-\frac{d^2u}{dx^2} + a^2u = f(x)$.

(3) Define Fourier sine and cosine transforms. Find Fourier sine and cosine transform of $f(x) = xe^{-x}$.

4 Answer the following : (any **two**) 7

(1) Define Mellin Tranform and its Inverse. If $f(x) = e^{-nx}$ then show that $M\left[e^{-nx}\right] = \frac{\Gamma p}{n^p}$.

(2) If $M\{f(x)\} = \overline{f}(p)$ then show that

$M\left\{\frac{1}{x} f\left(\frac{1}{x}\right)\right\} = \overline{f}(1-p)$ and

$M\left\{f(x^a)\right\} = \frac{1}{a}\overline{f}\left(\frac{p}{a}\right)$.

(3) If $f(x) = \frac{1}{e^{x-1}}$ then find $M(f(x))$.

5 Answer the following : (any **two**) 7

(1) Find Laplace transform of $f(t) = t^5 e^{3t}$ and $f(t) = t \cosh (4t)$.

(2) Apply convolution theorem to evaluate

$$L^{-1}\left\{\frac{1}{(s^2+1^2)(s^2+3^2)}\right\}.$$

(3) Derive Fourier Transform if $f(x)$ is even function. Also derive the result when $f(x)$ is odd function.