ACD-3859

Seat No. ________

M. Sc. (Sem. II) Examination

March / April – 2019

Mathematics : Paper-MTHP-IV

(General Topology)

Time : **3** Hours] [Total Marks : **90**

Instructions : (i) All questions are compulsory and carry equal marks.

(ii) Standard natations and convention are followed.

1 (a) Let X be an infinite set and $\tau = \{U \subset X / X - U$ is finite or X$\}$. Show that τ is a topology on X.

(b) Define a basis β for a topology on X.
Show that $\tau = \{U \subset X$ /for each $x \in U\ \exists B \in \beta \ni x \in B \subset U\}$ is a topology generated by basis β.

(c) If $\{\tau_\alpha\}$ be a family of topologies on X. Show that there is a unique largest topology on X contained in all τ_α.

If $X = \{a,b,c\}$ and

$\tau_1 = \{\phi, X, \{a\}, \{a,b\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b,c\}\}$ then find

(i) largest topology contained in τ_1

(ii) smallest topology containing τ_2.

OR

1 (a) Define a subbasis for a topology on X.
How would you generated the topology from a subbasis ? Justify your answer.

(b) Let R be the set of all real numbers and let $K = \{l/n | n$ is a natural number$\}$

Let $B = \{U : U = (a,b)$ or $U = (a,b) - k\}$. Generate a topology R_K from basis B. Show that R_K is strictly finer than the standard topology on R.

(c) If $\{\tau_\alpha\}$ be a family of topologies on X. Show that there is a unique smallest topology on X containing all the collection τ_α.

If $X = \{a,b,c\}$ and

$\tau_1 = \{\phi, X, \{a\}, \{a,b\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b,c\}\}$ then find

(i) largest topology contained in τ_1

(ii) smallest topology containing τ_2.

2 (a) Let $f : X \rightarrow Y$ be a mapping of one topological space into another. Show that f is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in X.

(b) Define Int(A); interior of a set, Bd(A); boundary of a set A, cl(D); closure of a set A.

Show that

(i) $\text{Int}(A) \cap \text{Bd}(A) = \Phi$

(ii) $\text{Int}(A) \cup \text{Bd}(A) = A$

(iii) For ant set A and D, $A \cap \text{cl}(D) \cup \text{cl}(A \cap D)$ if A is an open set.

(c) Give an example of a metric such that the metric topology induced by it is same as the usual topology on R. Compare that metric with the order topology on R.

OR

2 (a) Show that the order topology on Z_+ is the Discrete topology.

(b) Is the function $f:[0,1)\to\{(x,y)\in R^2 / x^2+y^2=1\}$ defined by $f(s)=(\cos 2\pi s, \sin 2\pi s)$ a homeomorphism ? Justify your answer.

(c) Let X be a topological space and $A\subset X$. If there is a sequence $\{X_n\}$ of points of A such that $x_n\to x$ in X then show that $x\in\overline{A}$. When does the converse true ? Justify.

3 (a) Show that finite product of a connected space is connected.

(b) Show that every path-connected space is connected. Is the converse true ? Justify your answer.

(c) Show that each path component of a topological space X lies in a component of X. If X is locally path connected then prove that both are the same.

OR

3 (a) Show that a space X is connected if and only if no proper subset of X is both closed and open. Deduce that R_l is disconnected.

(b) Show that connectedness is a topological property.

(c) If X has a discrete topology then show that X is totally disconnected. Is the converse true ? Justify your answer.

4 (a) State and prove that Lebesgue number lemma. Using this prove Uniform Continuity theorem.

(b) Show that every compact space is a limit point compact. Is the converse true ? Justify your answer.

(c) Is [0,1] as a subspace of R_l compact ? Is it a limit point compact ? Justify your answer.

OR

4 (a) State the tube lemma for a product space XxY. Show that the lemma does not hold if Y is not compact.

(b) Show that a compact subspace of a Hausdorff space is closed subspace.

(c) Show that a continuous one-one map from a compact space onto a Hausdorff space is a homeomorphism.

5 Attempt any six :

(a) Is the set $\{1, 1/2, 1/3, ...\} \cup \{0\}$,

(i) limit point compact

(ii) sequentially compact

(iii) compact and

(iv) locally compact in R ?

Justify your answer.

(b) The set $\{111, 222, 333, ..., 999\}$ is compact. Is the statement true ? Justify your answer. Also state at least two properties which the set does not possess.

(c) If $A = \left\{ m + \frac{1}{n} / m, n \in N \right\} \subseteq R$, find $\overline{A}$, with all details.

(d) Show that the components of Q are singletons. Show your detailed working.

(e) Is the finite complement topology a compact space ? Is a Hausdorff space ? Justify your both the answer with all details.

(f) Show that the Euclidean punctured space $R^n - \{0\}$ is connected.

(g) Give an example of space other than discrete whose components are singleton sets, with all details.

(h) Every discrete space is totally connected. Prove or give an example to disprove.

(i) Give an example of a locally connected space which is not connected. Justify.

(j) If a space A of the Euclidean metric space R^n is closed and bounded then show that it is compact.