ACA-3861

ACA-3861 Seat No._____

M. Sc. (Sem. II) Examination

March/April – 2019

Mathematics : Paper - MTHP-5

(Advanced Topology)

Time : 3 Hours] [Total Marks : 90

Instructions :

(1) All questions are compulsory.

(2) Standard notations and conventions are followed.

1 Answer the following : (any three) **18**

(1) Show that every metrizable space with countable dense subset has a countable basis.

(2) Show that $\mathbb{R}^I$ is Lindelöff space.

(3) For a continuous function $f : X \to Y$, show that X is Lindelöff then $F(X)$ is Lindelöff.

(4) Prove that if X is separable then every collection of disjoint open sets in X is countable.

2 Answer the following : (any three) **18**

(1) Prove that every compact Hausdorff space is normal.

(2) Prove that every regular Lindelöff space is normal.

(3) Show that every connected normal space having more than one point is uncountable.

(4) Prove that every well ordered set is normal in order topology.

3 Answer the following : (any two) **18**

(1) State and prove existence of finite partitions of unity.

(2) Let X be completely regular space. Show that X is connected if and only if $\beta(X)$ is connected.

(3) Prove that for a completely regular space X, there exists a compactification Y of X such that every bounded continuous function $f : X \to \mathbb{R}$ extends to a continuous function from Y to $\mathbb{R}$.

4 Answer the following : (any two) **18**

(1) Prove that a metric space (X, d) is compact if and only if it is complete and totally bounded.

(2) Show that if metric space (Y, d) is complete then the space $\left(Y^J, \overline{\rho}\right)$ is complete. Where $\overline{\rho}$ is uniform metric correspopnding to metric d.

(3) Prove that for matric space (X, d) there exists an isometric imbedding of X into a complete metric space.

5 Answer the following : (any six) **18**

(1) Give an example of Lindelöff space which is not second countable.

(2) Define completely regular space and prove that subspace of completely regular space is completely regular.

(3) Define Stone - Čech Compactification.

(4) Derive relation between uniform metric and sup metric.

(5) Is every bounded space totally bounded ?
Justify your answer.

(6) Prove Tietze extension theorem using Urysohn lemma.

(7) Prove that every normal space is regular space.

(8) Show that every finite set is closed in Hausdorff space.

(9) Give an example of Baire Space.