ACA-3880 Seat No._____

M. Sc. (Sem. II) Examination

March / April - 2019

Mathematics : MCB - II

(Advanced Linear Algebra)

Time : **2** Hours] [Total Marks : **35**

1 Attempt any **two** : **7**

(a) Let A be an algebra with unit element over F and suppose that A is dimension m over F. Then prove that every element in A satisfies some nontrivial polynomial in $F[x]$ of degree at most m.

(b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then prove that λ is a minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F.

(c) Explain the procedure to obtain $m_2(D)$ the matrix of linear transformation on the set of all polynomials in x of degree $n-1$ or less over field F.

2 Attempt any **two** : **7**

(a) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

(b) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F then show that T satisfies a polynomial of degree n over F.

(c) There exists a subspace W of V, invariant under T such that $V = V_1 \oplus W$.

3 Attempt any **two** : **7**

(a) State and prove Decomposition theorem.

(b) If $T \in A(V)$ then trace of T is the sum of the characteristic roots of T.

(c) If F is of characteristic 0 and if S and T in $A_F(V)$ are such that $ST - TS$ commutes then $ST - TS$ is nilpotent.

4 Attempt any **two** : **6**

(a) Define characteristic polynomial of T. Is every linear transformation T in $A_F(V)$ satisfies its characteristic polynomial ? Is every characteristics root of T is a root of product of elementary divisors. Justify your answer.

(b) The elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisor.

(c) For $A, B \in F_n$ prove that $\det(AB) = (\det A)(\det B)$.

5 Attempt the followings : **8**

(a) If V is finite-dimensional over F and if $T \in A(V)$ is invertible then T^{-1} is a polynomial expression in T over F.

(b) If $T \in A(V)$ is nilpotent then if $\alpha_0 + \alpha_1 T + ... + \alpha_m T^m$ is invertible if $\alpha_0 \neq 0$ where $\alpha_i \in F$.

(c) For $A, B \in F_n$ and $\lambda \in F$ then prove that $trace(AB) = trace(BA)$.

(d) State Cramer's rule.