AH-629 Seat No.________

B. Sc. (Sem. VI) Examination
March - 2019
Mathematics : CCMATH-602
(Mathematical Analysis - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Figures to the right side indicate marks of corresponding question.

1 (a) Suppose $E \subset X$, a matric space, p is a limit point of E, f and g are complex functions on E and $\lim_{x \to p} f(x) = A$, $\lim_{x \to p} g(x) = B$ then prove that **6**

$$\lim_{x \to p} \frac{f(x)}{g(x)} = \frac{A}{B} \text{ if } B \neq 0.$$

(b) State and prove Taylor's theorem. **6**

(c) Prove that continuous image of compact set in compact. **6**

OR

1 (a) Suppose X, Y, Z are matric spaces, $E \subset X$, functions $f : E \to Y$, $g : f(z) \to Z$, $h : E \to Z$ defined by $h(x) = g(f(x))$, $x \in E$. **6**

If f is continuous at point $P \in E$ and g is continuous at $f(p)$ then prove that h is continuous at p.

(b) State and prove L'HOSPITAL RULE. **6**

(c) State and prove Generalized Mean Value theorem. **6**

2 (a) If $\bar{F}:[a, b] \to R^k$ and $\bar{F} \in R(\alpha)$ where α is monotonically increasing then prove that **6**

(1) $|\bar{F}| \in R(\alpha)$ (2) $\left| \int_a^b \bar{F}\, d\alpha \right| \le \int_a^b |\bar{F}|\, d\alpha$

(b) Define : Least upper bound and Greatest Lower bound and also prove that **6**

$$L(p, f, \alpha) \le U(p, f, \alpha)$$

(c) $\left.\begin{aligned} f(x) &= 0, \ 0 \le x < 1 \\ &= 1, \ 1 \le x \le 2 \end{aligned}\right\}, \ \alpha(x) = f(x), \ x \in [0, 2]$ **6**

then using Defination of R-S prove that

$$f \notin R(\alpha)\,[a, b]$$

OR

2 (a) State and prove the Fundamental theorem of Calculus. **6**

(b) Suppose $C_n \ge 0,\ n = 1, 2, 3, \ldots, \sum C_n$ converges, $\{S_n\}$ is a sequence of distinct point in (a, b) and $d(x) = \sum_{n=1}^{\infty} C_n \cdot I(x - S_n)$, f is continuous on $[a, b)$ then prove that **6**

$$\int_a^b f\, d\alpha = \sum_{n=1}^{\infty} C_n \cdot f(S_n)$$

(c) Using Definition of R-S prove that **6**

$$\int_0^2 x\, d(x^2) = \frac{16}{3}$$

3 (a) State and prove the Cauchy criterion for uniform convergence. 6

(b) Suppose k is compact and 6

(1) $\{f_n\}$ is a sequence of continuous function on k.

(2) $\{f_n\}$ converges pointwise to a continuous function f on k.

(3) $f_n(x) \geq f_{n+1}(x) \ \forall \ x \in k, \ n = 1, 2, 3 \ldots\ldots$

then prove that $f_n \to f$ uniformly on k.

(c) Prove that $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ converges uniformally on [0, 1] and the term by term integration is possible for it. 6

OR

3 (a) Suppose uniformaly on a set E, Let x be a Limit point of E and $\lim_{t \to x} f_n(t) = A_n, (n = 1, 2, \ldots.)$ then prove that $\{A_n\}$ converges and 6

$$\lim_{t \to x} \lim_{n \to \infty} f_n(t) = \lim_{n \to \infty} \lim_{t \to x} f_n(t)$$

(b) Prove that $l(x)$ is a complete matric space. 6

(c) Let α be monotonically increasing on [a, b], suppose $f_n \in R(\alpha)[a, b]$, $n = 1, 2, \ldots..$, $f_n \to f$ uniformaly on $[a, b]$ then prove that 6

(1) $f \in R(\alpha)[a, b]$

(2) $\int_a^b f \, d\alpha = \lim_{n \to \infty} \int_a^b f_n \, d\alpha$

4 Attempt any two : 8

(a) Let f be defined on $[a, b]$ and f has a Local maximum at point $x \in (a, b)$ and $f'(x)$ exists then prove that $f'(x) = 0$.

(b) If $f_1 \in R(\alpha)[a, b]$, $f_2 \in R(\alpha)[a, b]$ then prove that $f_1 + f_2 \in R(\alpha)[a, b]$.

(c) State and prove M_n-test for uniform convergence.

5 Attempt any two : 8

(a) Discuss the type of discontinuity at $x = 0$,

Define $f(x) = \dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}$, $x \neq 0$, $f(0) = 0$.

(b) If $f(x) = [x]$, $\alpha(x) = x$ then prove that $f \in R(\alpha)[0, 2]$, $x \in [0, 2]$.

(c) Give an example of $\{f_n\}$ such that

$$\int_a^b \lim_{n\to\infty} f_n(x)\, dx \neq \lim_{n\to\infty} \int_a^b f_n(x)\, dx.$$